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A Case Study on Set Theory in Discrete Mathematics

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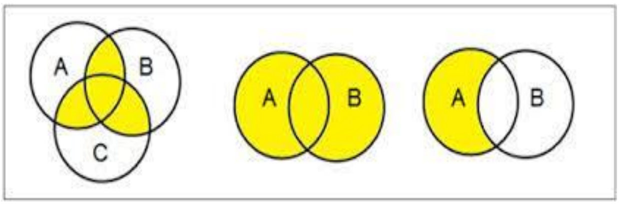
Abstract—**Set theory is branch of mathematical logic and foundation known as set theory. Set theory is branch of mathematical logic that studies set, which informally are collections of objects.** Set theory is important mainly because it serves as a foundation for the rest of mathematics--it provides the axioms from which the rest of mathematics is built up

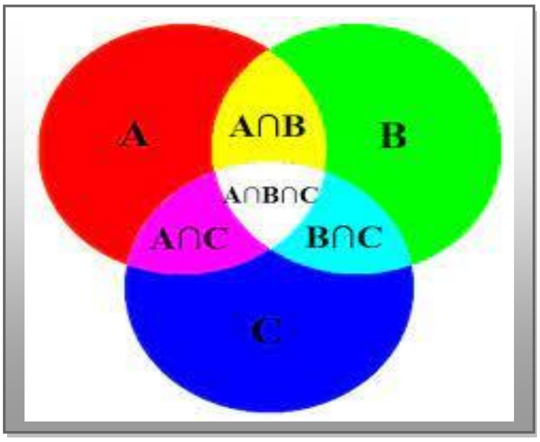


Index Terms—**Set theory, Universal Set, Null Set, Finite Set, Infinite Set, Equivalent Set, Venn diagram and De Morgan's law.**

I. Introduction

Set theory is the mathematical theory of well-determined collections, called sets, of objects that are called members, or elements, of the set. The axioms of set theory imply the existence of a set-theoretic universe so rich that all mathematical objects can be construed as sets.

Set theory is the mathematical theory of well-determined collections, called sets, of objects that are called members, or elements, of the set. Pure set theory deals exclusively with sets, so the only sets under consideration are those whose members are also sets.



Set: Set is an methodical structure which on be defined as the well defined

Unordered of distinct objects A set is a collection of distinct objects. This means that {a, b, c} is a set but {a, b, c} is not because 'a' appears twice in the second collection. The second collection is called a multiset. Sets are often specified with curly brace notation. The notation a ∈ A denotes that 'a' is an element of the set A. If 'a' is not a member of A, write a ∈ / A.

Equal set: Two or more sets are said to be equal sets if they have same elements

S1= {a,b,c} , S2={c,b,a}

When two sets have the same and equal elements, they are called Equal Sets. The arrangement or the order of the elements does not matter, only

The same elements in each set matter

Empty set: A set is said to be empty set if it doesn't have any element S= { }

Any Set that does not contain any element is called the empty or null or void set. The symbol used to represent an empty set is – {} or φ. Examples: Let A = {x: 9 < x < 10, x is a natural number}

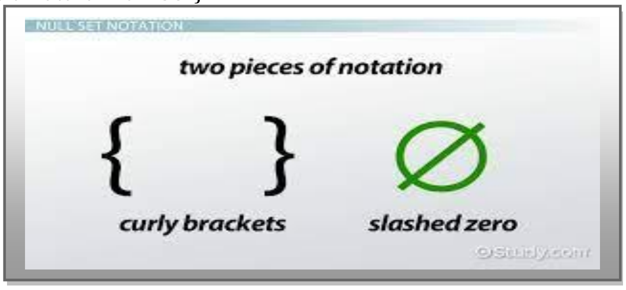


Fig. 1: Set Notation

Representation of set:

1. Roster method (tabular or listing method)
2. Set builder (property method)
3. Graphical method

Roster method:

- The roster method is defined as a way to show the elements of a set by listing the elements inside of brackets
- We list all elements of set in braces if possible
- X={a1,a2,a3,a4}
- Ordered doesn't matter in set
- Reputation is there the elements is consider only once
- X ={a,a,b,c,c} = {a,b,c}

Set builder method:

Specify the property (or properties) that all members of the set must satisfy.

S = {x | x is a positive integer less than 100}

S = {x | x ∈ Z + ∧ x < 100}

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$S = \{x \in Z+ \mid x < 100\}$

Graphical method:

A **set** can be represented **graphically** by a closed geometric shape. There are several closed geometric shapes in geometry. Hence, a **set** can be represented by any one of the closed geometric shapes. For example, a circle, a triangle, a square, or any other closed geometric shape

Cardinality of set:

A **set** A. If A has only a finite number of elements, its **cardinality** is simply the number of elements in A. For example, if

A= {2, 4, 6, 8, 10}, then |A|=5.

Definition

If there are exactly n distinct elements in a set A, where n is a nonnegative integer, we say that A is finite. Otherwise it is infinite

Definition

The cardinality of a finite set A, denoted by |A|, is the number of (distinct) elements of A.

| {1, 2, 3}| = 3

Note: 1. Cardinality of a set can never be fractional, negative

2. Cardinality of set is always a non-negative

Equivalent sets:

Two sets are said to be equivalent if they have same cardinality

X= {a, b, c} |X|=3

Y= {1, 2, 3} |Y|=3

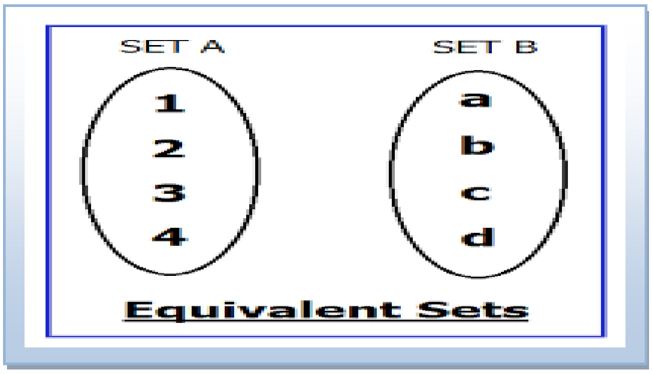


Fig. 2: Equivalent Set

Subset of set:

A set X is said to subset of a set Y if all the elements of X in Y

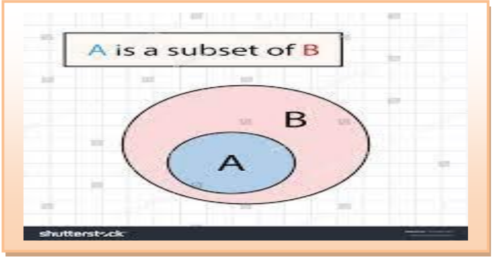
X= {a, b, c} Y= {a, b, c, d} : X ⊆ Y

X is subset of Y

Or

Set Y include X

Total possible subsets of set =2^n : where n=cardinality



Proper Subset:

Set X is a proper subset of Y if X ⊆ Y and X /= Y.

This is denoted by X ⊂Y

X= {a, b, c} Y= {a, b, c, d}

X ⊂Y

Power set:

The set of all subsets of a set S is called the power set of S. It is denoted by P(S) or 2S.

Formally: P(S) = {S 0 | S 0 ⊆ S}

Power set of a set A is a set contain all the possible subset of set A

It is denoted by P (A) or 2^A

A= {a, b}

P (A) = {{ }, {a}, {b},{a, b}}

*cardinality of power set of A =total possible subsets of A

|P (A)|=2^n

II. Set Operations:

Union:

The union of two sets A, B is defined by A ∪ B = {x | x ∈ A ∨ x ∈ B}

A= {1, 2, 3}

B= {2, 3, 4}

A ∪ B = {1, 2, 3, 4}

General union of several sets: A1 ∪ · · · ∪ An = {x | x ∈ A1 ∨ · · · ∨ x ∈ an}

Intersection:

The intersection of two sets A, B is defined by A ∩ B = {x | x ∈ A ∧ x ∈ B}

General intersection of several sets: A1 ∩ · · · ∩ An = {x | x ∈ A1 ∧ · · · ∧ x ∈ An}

A= {a, b, c}

B= {a, c, d}

A ∩ B = {a, c}

Complement:

The complement of A w.r.t. U is defined by A = {x ∈ U | x ∈/ A}

Disjoint sets and collection of sets

• **Disjoint sets**: Two sets A and B are called disjoint iff A∩B = ϕ, i.e., A and B have no element in common.

• A collection of sets (set of sets) is called a disjoint collection if, for every pair of sets in the collection,

The two sets are disjoint. The elements of a disjoint collection are said to be mutually disjoint.

Let A be an indexed set A = {Ai}i∈J. The set A is a disjoint collection

If Ai ∩ Aj = ϕ for all i, j∈J, i≠j.

Set difference:
The difference between sets A and B, denoted A − B is the set containing the elements of A that are not in B. Formally: A − B = {x | x ∈ A ∧ x ∈/ B} = A ∩ B

Symmetric difference:
The symmetric difference between sets A and B, denoted A4B is the set containing the elements of A that are not in B or vice-versa. Formally: A4B = {x | x ∈ A xor x ∈ B} = (A − B) ∪ (B − A)
AxorB = (A ∪ B) − (A ∩ B)

Relative Complement
• Let A and B be any two sets. The relative complement of B in A (or of B with respect to A), written as
A – B or A\B is the set consisting of all elements of A which are not elements of B.
A – B = {x | xεA ∧ x ∉B}
• The relative complement of B in A is also called the difference of A and B.

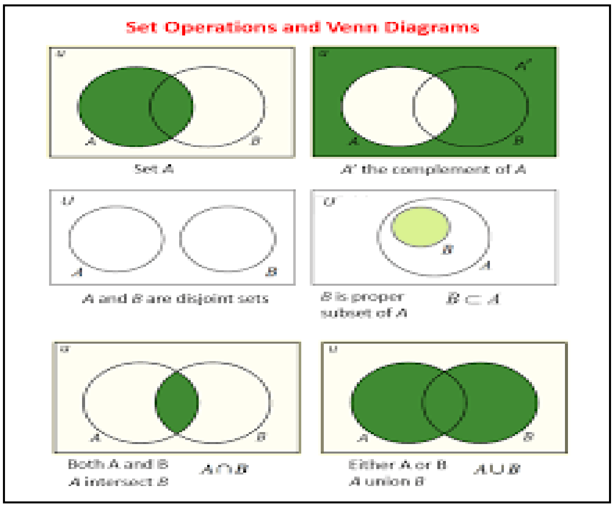


Fig. 3: Set Operations and Venn diagram

Cartesian product of a set with itself
Cartesian product A×A is also written as A2 and similarly A×A×A as A3 and so on.
A2 is called the Cartesian square of set X
A3 is called the Cartesian cube of set X
An is called the n-ary Cartesian power of set X

Set Identities

- The basic method to prove a set identity is the element method or the method of double inclusion. It is based on the set equality definition: **two sets and are said to be equal if any**. In this method, we need to prove that the left-hand side of a set identity is a subset of the right-hand side and vice versa.

Check for Associativity
• (A×B) ×C = {((a, b), c) | ((a, b)εA×B) ∧ (cεC) } = { (a, b, c) |
(aεA) ∧ (bεB) ∧ (cεC)}
• A× (B×C) = {(a, (b, c)) | (aεA) ∧ ((b, c)ε B×C) }
Here (a, (b, c)) is not an ordered triplet.

(A×B) ×C contains ordered pairs in which the first member is an ordered pair and second member is
an element of C(it is a ordered triplet).
A× (B×C) contains ordered pairs in which the first member is an element of A while the second
Member is an ordered pair.

• (A×B) ×C ≠ A× (B×C)

TABLE 1 Set Identities.

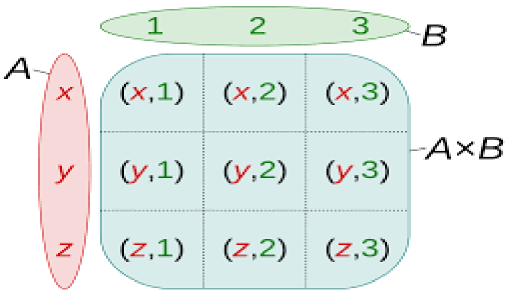
Identity	Name
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

Table. 1: Set Identifiers

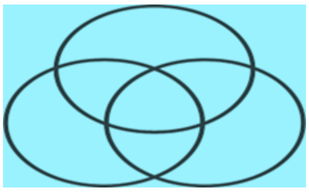
Cartesian product:
A new set can be constructed by associating every element of one set with every element of another set. The Cartesian product of two sets A and B, denoted by A × B is the set of all ordered pairs (a, b) such that a is a member of A and b is a member of B
{1, 2} × {1, 2} = {(1, 1), (1, 2), (2, 1), (2, 2)}.
{a, b, c} × {d, e, f} = {(a, d), (a, e), (a, f), (b, d), (b, e), (b, f), (c, d), (c, e), (c, f)}. .

Some basic properties of Cartesian products:
A × ∅ = ∅
A × (B ∪ C) = (A × B) ∪ (A × C)
(A ∪ B) × C = (A × C) ∪ (B × C)

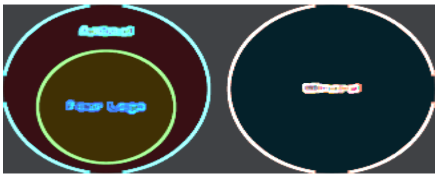
Let A and B be finite sets; then the cardinality of the Cartesian product is the product of the cardinalities:
|AxB|=|BxA



Venn diagram:
It represent all possible between given sets
# of region= 2^n
Where n is # sets under study except univarsall set
Set - closed curves
nC0 + nC 1 +nC2 +...........+nCn=2^n



Euler daigram:
Its represent only those relation exist in given sets
Region depend upon relationship exist between given Sets
Set closed curves
Region max=2^n
#region min =n+1



Set under study except universal set

III. CONCLUSION

The theory of sets has been the base for the foundation of mathematics and so is considered as one of the most significant branches in mathematics. The fact that any mathematical concept can be interpreted with the help of set theory has not only increased its versatility but has established this theory to be the universal language of mathematics. In the recent past, a relook to the concept of uncertainty in science and mathematics has brought in paradigmatic changes. Prof. Zadeh, through his classical paper, introduced the concept of modified set called fuzzy set to be used a mathematical tool to handle different types of uncertainty with the help of linguistic variable

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