https://doi.org/10.31871/WJRR.14.5.7

World Journal of Research and Review (WJRR)
ISSN: 2455-3956, Volume-14, Issue-5, May 2022 Pages 01-14

A Modified Goodness of Fit Tests for Type 2 Censored Sample from Normal Population

Hala .M. A. Aboalliel

***Abstract*— In this paper we consider nonparametric procedure for assessing a new modified goodness of fit technique for the normal distribution from type 2 censored samples. Sample sizes are chosen 10(10) 60 censored at the order statistic, r=0.6n. Goodness of fit tests based on the empirical distribution function are used, which are the Cramer von Mises CvM test and the Anderson Darling AD test. The critical values for the tests are generated. The power of the tests for various alternative distributions is computed.**

***Index Terms*— Goodness, nonparametric.**

I. INTRODUCTION

An important problem in statistics is to find information about the form of the population from which a sample is drawn. Goodness of fit tests are given for the normal distribution with unknown mean and unknown variance from type 2 censored samples. We use the modified Cramer-von Mises (CvM) and Anderson-Darling (AD) goodness of-fit tests.

A Monte Carlo procedure is used to develop and compare the modified goodness-of-fit (GOF) tests from censored samples. Critical values for different sample sizes n are generated. Sample sizes are chosen 10(5)60 (i.e. sample sizes started at n=10 and ends at n=60 with a step of 5) censored at the r^{th} order statistics where $r = 0.6 \times n$ for example at sample size 25, r=15.The modified CvM and AD test statistics are calculated for the given values of n, this procedure is repeated 10000 times for each test statistic. These 10000 values are then ranked, and we find the 80%, 85%, 90%, 95%, and 99% quantiles. These quantiles approximate the critical values for respective significance levels of 0.20, 0.15, 0.10, 0.05, and 0.10, for each test. Tables of critical values, for the two modified test statistics from type 2 censored samples for the normal model are found. Also the power study of the modified tests to compare the efficiency of the CvM and AD tests under different conditions is discussed.

II. THE NORMAL DISTRIBUTION

The normal distribution is without a doubt the most important and most widely used continuous probability distribution. It is the fundamental bases of the application of statistical inference in analysis of data, because the distributions of several important sample statistics tend toward a normal distribution as the sample sizes increases.

The probability density function of the normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} ,$$

$$-\infty < x < \infty \ , \ -\infty < \mu < \infty \ , \ \sigma. > 0 \qquad (2\text{-}1)$$

the parameters μ *and* σ^2 are the mean and variance, respectively, of the normal random variable X.

If we have the ordered observations $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ of a normal random sample, and some of these observation are missing, the sample is said to be censored. At the r^{th} order statistics, if all the observations less than $x_{(r)}$ are missing, then the sample is left-censored or type 1 censoring, and if all the observations greater than $x_{(r)}$ are missing, it is right -censored or type2 censoring. We consider the case of type 2 censoring i.e. if $x_{(1)} < x_{(2)} < \dots < x_{(r)}$ be the available observations in a sample of size n.

Maximum likelihood estimates are complicated to calculate and percentage points of the test statistics for finite n appear to converge more slowly to the asymptotic points when these estimates are used (D'Agostino and Stephens 1986**)**. Gupta (1952**)** suggested estimates of μ and σ which we used here, these are linear combinations of the available order statistics

$$\mu^* = \sum_{i=1}^{r} b_i x_{(i)} \ ,$$

and $$\sigma^* = \sum_{i=1}^{r} c_i x_{(i)} \qquad (2\text{-}2)$$

Hala .M. A. Aboalliel, Qassim University, Saudi Arabia

where $b_i = \frac{1}{r} - \frac{\overline{m}(m_i - \overline{m})}{\sum_{i=1}^{r}(m_i - \overline{m})^2}$, $c_i = \frac{m_i - \overline{m}}{\sum_{i=1}^{r}(m_i - \overline{m})^2}$

and m_i is the expected value of the i-th order statistic of a sample of size n from the standard normal distribution and where $\overline{m} = \sum_{i=1}^{r} \frac{m_i}{r}$. Values of m_i are tabulated or can be well approximated by Blom(1958):

$$m_i = \Phi^{-1}\left(\frac{i - 0.375}{n - 0.125}\right) \qquad (2\text{-}3)$$

where $\Phi^{-1}(.)$ the inverse C.D.F of the standard normal, and the estimates μ^*, σ^* have been shown to be asymptotically efficient (Ali and Chan 1964). These estimates are the same as those obtained by least squares when x_i is regressed against $m_i, i = 1,\ldots,r$ (D'Agostino and Stephens 1986).

III. Goodness Of Fit Test Statistics For Type 2 Censored Data

Goodness-of-fit tests (GOF) measure the degree of agreement between the distribution of an observed data sample and the theoretical statistical distribution.

A goodness of fit test based on the empirical distribution function (E.D.F), where the parameters are estimated is called a modified goodness of fit test.

E.D.F statistics are based on the vertical differences between the empirical distribution function $F_n(x)$ and the theoretical statistical distribution $F(x)$ and they are divided into two classes, the supremum class and the quadratic class.

The supremum statistics class:

This includes, D^+ and D^- defined as:

$$D^+ = sup_x\{F_n(x) - F(x)\}, \text{ and } D^- = sup_x\{F(x) - F_n(x)\}.$$

the most well-known EDF statistic is D, introduced by Kolmogrov (1933):

$$D = sup_x|F_n(x) - F(x)| = max(D^+, D^-).$$

A closely related statistic V, given by Kuiper (1960),which is defined by:

$$V = D^+ + D^-$$

The quadratic statistics class:

A second and wide class of measures of discrepancy is given by the Cramer-von Mises family

$$Q = n\int_{-\infty}^{\infty}\{F_n(x) - F(x)\}^2 \Psi(x) dF(x) \qquad (3\text{-}1)$$

where $\Psi(x)$ is a suitable function, which gives weights to the squared difference $\{F_n(x) - F(x)\}^2$. When $\Psi(x) = 1$ the statistic is the Cramer-von Mises statistic CvM denoted by W^2, and when $\Psi(x) = [\{F(x)\}\{(1 - F(x))\}]^{-1}$ the statistic is the Anderson-Darling

We will define the goodness of fit test statistics for type 2 censored data. First for a specified sample of the normal population of size n, we have the ordered statistics $x_{(1)} < x_{(2)} < \ldots < x_{(r)}$ and suppose the distribution of *x* is *F*(*x*), based on these sample the probability integral transformation be defined as follows:

$$z_{(i)} = F(x_{(i)}) \qquad (3\text{-}2)$$

which is itself censored i.e. $z_{(1)} < z_{(2)} < \ldots < z_{(r)}$ with $z_{(r)}$ the largest and *r* fixed.

Then the Kolmogrov-Smirnov(K-S) and Cramer-von Mises (CvM) statistic tests for type 2 censoring defined as the following:

1- The Kolmogrov-Smirnov statistics D^+, D^-, and D:

The Kolmogrov-Smirnov statistic, modified for type 2 censored data is ${}_2D_{r,n}$, calculated from the EDF $F_n(z)$ of the ordered z-values:

$$\begin{aligned} {}_2D_{r,n} &= \sup_{0\le z\le z_{(r)}} |F_n(z) - z| \\ &= \max_{1\le i\le r}(D^+, D^-) \\ &= \max_{1\le i\le r}\left\{\frac{i}{n} - z_{(i)}, z_{(i)} - \frac{i-1}{n}\right\} \\ &= \max_{1\le i\le r}\left|\frac{i-0.5}{n} - z_{(i)}\right| + \frac{0.5}{n} \end{aligned}$$

2- Cramer-von Mises Statistics:

A second group of statistic for censored samples is the general Cramer-von Mises type. Pettitt and Stephens (1975**)** introduced versions of the Cramer-von Mises ${}_2W^2_{r,n}$, Watson ${}_2U^2_{r,n}$ and Anderson-Darling ${}_2A^2_{r,n}$ statistics, obtained for type 2 censored data by modifying the upper limit of integration in the definition of these statistics, given $z_{(1)} < z_{(2)}, \ldots < z_{(r)}$ the formulas are

$${}_2W^2_{r,n} = \sum_{i=1}^{r}\left(z_{(i)} - \frac{2i-1}{2n}\right)^2 + \frac{r}{12n^2} + \frac{n}{3}\left(z_{(r)} - \frac{r}{n}\right)^3, \qquad (3\text{-}3)$$

$$\begin{aligned} {}_2A^2_{r,n} = &-\frac{1}{n}\sum_{i=1}^{r}(2i-1)\left[\log z_{(i)} - \log\{1 - z_{(i)}\}\right] - 2\sum_{i=1}^{r}\log\{1 - z_{(i)}\} \\ &-\frac{1}{n}\left[(r-1)^2\log\{1 - z_{(i)}\} - r^2\log z_{(r)} + n^2 z_{(r)}\right] \end{aligned} \qquad (3\text{-}4)$$

$${}_2U^2_{r,n} = {}_2W^2_{r,n} - nz_{(r)}\left[\frac{r}{n} - \frac{z_{(r)}}{2} - \frac{r\bar{z}}{nz_{(r)}}\right]^2 \qquad (3\text{-}5)$$

where $\bar{z} = \sum_{i=1}^{r} z_{(i)}/r;$

To this point, an approach depending on the replacement of the E.D.F by the nonparametric denstiy is used. In equations (3-3) and (3-4) the expression of the EDF will be replaced by

$$\hat{F}(x) = \frac{1}{n}\sum_{i=1}^{n}\Phi\left(\frac{x - x_i}{h}\right) \qquad (3\text{-}6)$$

where $\hat{F}(x)$ is the cumulative distribution function C.D.F of the nonparametric kernel estimator $\hat{f}(x)$ for the normal density $f(x)$, Φ(x) denotes the C.D.F. for the standard normal distribution, and h is the bandwidth and will be take h=1.06 (Silverman 1986). The kernel estimator based on a random sample $x_1, x_2, \ldots, x_n$ from the normal population with density function $f(x)$ is defined as:

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n}K\left(\frac{x - x_i}{h}\right) \qquad \forall\ x \in R \qquad (3\text{-}7)$$

where the kernel function K is a symmetric probability density function on the entire real line. We use the nonparametric kernel estimator with the Gaussian kernel, which will be defined as:

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n}\frac{1}{\sqrt{2\pi}}\exp\left(\frac{x-x_i}{h}\right)^2 \qquad (3\text{-}8)$$

IV. CRITICAL VALUES

Critical values for the modified goodness-of-fit tests are generated using Monte Carlo procedure. Lilliefors (1966**)** first used this approach to find tables of critical values for a modified Kolmogrov-Smirnov (K-S) test for the normal distribution with estimated mean and variance. And for the exponential distribution with unknown mean, Lilliefors (1967**)** were introduced with a study of the power of the test, which showed that the modified K-S test had higher power than χ^2 -test for the normal case.

Tables of critical values for the modified K-S, CvM and AD statistics using Monte Carlo(M.C) techniques for the extreme value distribution where the MLE for the parameters is used are derived in a paper by Littele et al**.** in (1979).

Tables for the percentage points for the modified K-S, AD and CvM statistics for the gamma distribution are derived in Woodruff et al., **(**1984**)**.

Modified goodness of fit testes for the inverse Gaussian distribution were done by Gunes, Dennis Dietz, Auclair and Moore (1997**)**, also a modification of the Kolmogrov-Smirnov test for the Erlang-2 distribution is given by Neil B Mrks (1998**)**. Choulakian, Lockhart and Stephens(1994) developed the Cramer-von Mises statistics for use in testing the discrete distribution and gives tables for tests for the discrete uniform distribution.

The Technique And The Result

The Monte Carlo procedure for this application was divided to 3 basic stages, the first stage contain the critical values determination, the second stage determine the power performance of the two modified test statistics, and the third stage contain the power comparison.

Stage I

Critical values determination:

The procedure is as follows: Suppose the sample is censored (type 2), and H_0 is

H_0: the censored sample $x_{(1)} < x_{(2)} ... < x_{(r)}$ comes from the normal distribution $F(x)$, with unknown mean and unknown variance.

The following steps are performed for the null hypothesis:

Step 1

A sample of n normal random varieties $x_1, x_2, ..., x_n$ is generated from the normal distribution with mean 100 and variance 10, n takes the values 10(5) 60 using the RNNOR routine from IMSL library with initial seed to generate the uniform random number.

Step 2

The random varieties are converted to order statistics by sorting them in ascending ordered, then the ordered sample is censored at the r^{th} order statistic, where $r = 0.6 \times n$

Step 3

The normal parameters μ^* and σ^* are obtained using equation (2-2) .

Step 4

Find $w_i = \{x_{(i)} - \mu^*\}/\sigma^*$, $i = 1,2,...r$.

Step 5

Calculate $z_{(i)} = \Phi(w_i)$ where $\Phi(.)$ is the standard normal C.D.F.

Step 6

Find a continuous nonparametric fit $\hat{F}(x)$ as in equation (3-6).

Step 7

The modified ${}_2W^2_{r,n}$, ${}_2A^2_{r,n}$ test statistics are calculated by substituting the $z_{(i)}$, $i = 1,2,...r$ values and the nonparametric fit $\hat{F}(x)$ in place of the E.D.F in equations (3-3) and (3-4).

Step 8

Steps 1-7 are repeated 10000 times to generate 10000 independent test statistics for each type.

Step 8

For each type of test, the 10000 statistic are ordered. We find the percentiles $80^{th}, 85^{th}, 90^{th}, 95^{th}$ and 99^{th}, the pth percentile is defined by:

i- The (k+1)th largest sample point if $\left(\frac{10000p}{100}\right)$ is not an integer (where k is the largest integer less than $\left(\frac{10000p}{100}\right)$.

ii-The average of the $\left(\frac{10000p}{100}\right)th$ and $\left(\frac{10000p}{100}+1\right)th$ largest observations if $\left(\frac{10000p}{100}\right)$ is an integer. These percentiles approximate the critical values for respective significance levels α of *0.20, 0.15 ,0.10, 0.05 , 0.01*.

Stage II

The test performance under H_0 for the derived critical values:

The following steps are performed for the null hypothesis:

Step 1

A sample of n normal random varieties $x_1, x_2, \ldots, x_n$ is generated from the normal distribution with mean 100 and variance 10, n takes the values 10(5) 60 using the RNNOR routine from IMSL library with different seed to generate the uniform random number.

Step 2

The null hypothesis H_0 is assumed and steps 2-7 of the critical value generation procedure are performed to compute values for the CvM and AD test statistics.

Step 3

The corresponding power study for the hypothesis is conducting under H_0 and the power is computed. The test shows powers, which were reasonably close to the α-levels.

Stage III

Power Comparison:

The power of a statistical test is the probability of correctly rejecting a false null hypothesis. In our case, the null hypotheses (H_0) is the censored sample $x_{(1)} < x_{(2)} < \ldots < x_{(r)}$ comes from the normal distribution. The alternative hypothesis (H_a) is that the sample follows some other distribution. The following alternative distributions are considered:

- H_1 : Uniform over the range 0.0 to 1.0
- H_2 : Chi square with 1 degree of freedom
- H_3 : Chi square with 4 degree of freedom
- H_4 : Negative Exponential
- H_5 : Cauchy
- H_6 : Double Exponential
- H_7 : t-student distribution with 3 d.f.
- H_8 : Logistic distribution
- H_9 : Normal distribution .

The sample size *n* is varied from 10 to 60 with increments of 5 censored at r^{th} order statistics such that $r = 0.6 \times n$, and the significance levels α, again include 0.20, 0.15, 0.10, 0.05 and 0.01. For each distribution.

The following steps are performed:

Step1

A sample of n random varieties is generated from the selected alternative distributions.

Step 2

The null hypothesis H_0 is assumed and steps 2-7 of the critical value generation procedure are performed to compute values for the CvM and AD test statistics.

Step 3

For the given distribution and significance level α, H_0 is rejected if the test statistic exceeds the corresponding critical value.

Step 4

Steps 1-3 are repeated 10000 times to generate 10000 independent sets of test statistic values.

Step 5

The power of each test is obtained by counting the number of times H_0 is rejected and dividing by 10000.

Results

The results are shown in the following tables. The tables give the critical values for both cases when the CvM statistic is used and when the AD statistic is used. The tables also show the power of both tests for different sample sizes.

Thus this application defines a new modified goodness of fit test, both the CvM and AD statistics are used. The critical values are derived by Monte Carlo experiment. Then the power of the test for the case of the CvM and AD is obtained when the underlying distribution is normal. This power shows a value which is close to the significance level. The test is then performed against each of the nine different alternatives. The power for the different distributions using the modified CvM and AD statistics shows an increasing power when the sample size increase the two tests discriminates all other distributions with high powers except for logistic and the normal, the modified test using the AD statistic gives better power than the test based on the CvM statistic for different alternatives.

Critical Values for the New Suggested Test for Censored Normal at Censored ratio 0.6 with Sample Size = 10 (5) 60 (Using CvM) (at Significance Levels .2,.15,.1,.05,.01)

N	0.20	0.15	0.10	0.05	.01
10	0.0285	0.0328	0.0392	0.0536	0.0958
15	0.0311	0.0355	0.0430	0.0590	0.1115
20	0.0326	0.0375	0.0447	0.0601	0.1201
25	0.0328	0.0378	0.0453	0.0599	0.1102
30	0.0337	0.0385	0.0458	0.0603	0.1080
35	0.0339	0.0388	0.0459	0.0603	0.1018
40	0.0341	0.0390	0.0461	0.0606	0.0984
45	0.0342	0.0392	0.0461	0.0599	0.1038
50	0.0342	0.0389	0.0469	0.0606	0.1054
55	0.0348	0.0398	0.0460	0.0607	0.1044
60	0.0345	0.0392	0.0463	0.0597	0.1006

Power of Tests for the Censord Normal at Censord ratio 0.6 for Sample Size = 10 (5) 60 (Using CvM) (at Significance Levels .2,.15,.1,.05,.01)

N	0.20	0.15	0.10	0.05	.01
10	0.2096	0.1533	0.1064	0.0534	0.0139
15	0.2030	0.1550	0.1006	0.0479	0.0111
20	0.1980	0.1490	0.0998	0.0499	0.0080
25	0.2035	0.1486	0.0992	0.0488	0.0092
30	0.1996	0.1527	0.1044	0.0535	0.0096
35	0.2023	0.1483	0.1039	0.0500	0.0118
40	0.1980	0.1487	0.0994	0.0503	0.0109
45	0.2014	0.1474	0.1002	0.0498	0.0121
50	0.2084	0.1565	0.0989	0.0491	0.0101
55	0.2015	0.1514	0.1038	0.0465	0.0082
60	0.2030	0.1522	0.1015	0.0517	0.0099

Power of the test for censored normal with sample 10 censored at 6 (Using CvM)
(Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.3312	0.6956	0.3571	0.4836	0.4097
.15	0.2690	0.6454	0.2959	0.4210	0.3380
.10	0.2016	0.5696	0.2265	0.3419	0.2569
.05	0.1182	0.4488	0.1410	0.2334	0.1522
.01	0.0345	0.2357	0.0435	0.0903	0.0039

Sign. Level	D.E.	t(3)	Logistic	Normal
.20	0.2108	0.2222	0.2014	0.2004
.15	0.1511	0.1626	0.1450	0.1500
.10	0.0898	0.1059	0.0927	0.1002
.05	0.0312	0.0451	0.0382	0.0500
.01	0.0052	0.0071	0.0084	0.0101

Unif.=Uniform Ch(k)=Chi square with k d.f Exp.=Negative Exponential D.E.=Double Exponential t(3)=t-student distribution with 3 d.f.

Power of the test for censored normal with sample 15 censored at 9 (Using CvM) (Normal against one of the following:)

Sign.Lelvel	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.4082	0.8625	0.4239	0.6272	0.5603
.15	0.3448	0.8292	0.3673	0.5734	0.5041
.10	0.2673	0.7746	0.2909	0.4932	0.4251
.05	0.1668	0.6718	0.1922	0.3664	0.2906
.01	0.0528	0.4208	0.0619	0.1542	0.0767

Sign. Level	D.E.	t(3)	Logistic	Normal
.20	0.2274	0.2600	0.1972	0.2001
.15	0.1676	0.2018	0.1470	0.1501
.10	0.1002	0.1364	0.0888	0.1001
.05	0.0355	0.0594	0.0394	0.0501
.01	0.0028	0.0070	0.0065	0.0100

Power of the test for censored normal with sample 20 censored at 12 (Using CvM) (Normal against one of the following:)

Sign.Lelvel	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.4821	0.9377	0.4986	0.7321	0.6791
.15	0.4137	0.9174	0.4387	0.6775	0.6228
.10	0.3359	0.8879	0.3628	0.6075	0.5550
.05	0.2326	0.8208	0.2539	0.4875	0.4302
.01	0.0698	0.5876	0.0804	0.2174	0.1518

Sign. Level	D.E.	t(3)	Logistic	Normal
.20	0.2559	0.2842	0.1997	0.2001
.15	0.1868	0.2255	0.1465	0.1501
.10	0.1219	0.1605	0.0928	0.1002
.05	0.0480	0.0800	0.0410	0.0501
.01	0.0015	0.0074	0.0046	0.0100

Power of the test for censored normal with sample 25 censored at 15 (Using CvM) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.5630	0.9756	0.5769	0.8234	0.7697
.15	0.4965	0.9658	0.5110	0.7779	0.7259
.10	0.4185	0.9502	0.4340	0.7164	0.6670
.05	0.2982	0.9143	0.3202	0.6127	0.5615
.01	0.1111	0.7630	0.1285	0.3513	0.2967

Sign. Level	D.E.	t(3)	logistic	Normal
.20	0.3004	0.3375	0.2124	0.1999
.15	0.2299	0.2710	0.1530	0.1501
.10	0.1528	0.1949	0.0957	0.1001
.05	0.0662	0.1091	0.0434	0.0502
.01	0.0035	0.0174	0.0051	0.0101

Power of the test for censored normal with sample 30 censored at 18 (Using CvM) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.6189	0.9895	0.6273	0.8779	0.8284
.15	0.5579	0.9865	0.5707	0.8459	0.7972
.10	0.4777	0.9795	0.4955	0.8008	0.7508
.05	0.3561	0.9595	0.3741	0.7113	0.6592
.01	0.1527	0.8683	0.1687	0.4590	0.4047

Sign. Level	D.E.	t(3)	Logistic	Normal
.20	0.3285	0.3705	0.2075	0.2001
.15	0.2609	0.3084	0.1530	0.1499
.10	0.1800	0.2336	0.1019	0.1002
.05	0.0843	0.1429	0.0458	0.0501
.01	0.0052	0.0311	0.0060	0.0100

Power of the test for censored normal with sample 35 censored at 21 (Using CvM) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.6871	0.9963	0.6766	0.9177	0.8825
.15	0.6291	0.9942	0.6225	0.8938	0.8555
.10	0.5505	0.9904	0.5517	0.8575	0.8177
.05	0.4149	0.9815	0.4321	0.7831	0.7384
.01	0.2074	0.9350	0.2225	0.5684	0.5315

Sign. Level	D.E.	t(3)	Logistic	Normal
.20	0.3720	0.4105	0.2186	0.2001
.15	0.2972	0.3426	0.1632	0.1500
.10	0.2172	0.2678	0.1055	0.1001
.05	0.1077	0.1669	0.0475	0.0501
.01	0.0118	0.0478	0.0059	0.0100

Power of the test for censored normal with sample 40 censored at 24 (Using CvM) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.7341	0.9988	0.7286	0.9441	0.9197
.15	0.6794	0.9982	0.6760	0.9246	0.9020
.10	0.6076	0.9971	0.6011	0.8963	0.8743
.05	0.4742	0.9918	0.4805	0.8360	0.8141
.01	0.2564	0.9667	0.2754	0.6650	0.6338

Sign. Level	D.E.	t(3)	Logistic	Normal
.20	0.4130	0.4460	0.2276	0.2001
.15	0.3351	0.3831	0.1720	0.1501
.10	0.2508	0.3109	0.1144	0.1002
.05	0.1317	0.2005	0.0503	0.0502
.01	0.0184	0.0693	0.0081	0.0101

Power of the test for censored normal with sample 45 censored at 27 (Using CvM) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.7816	0.9999	0.7667	0.9681	0.9413
.15	0.7292	0.9995	0.7201	0.9543	0.9281
.10	0.6608	0.9988	0.6576	0.9349	0.9077
.05	0.5389	0.9968	0.5454	0.8904	0.8591
.01	0.2735	0.9812	0.2944	0.7130	0.6792

Sign. Level	D.E.	t(3)	Logistic	Normal
.20	0.4574	0.4854	0.2404	0.2001
.15	0.3733	0.4212	0.1802	0.1502
.10	0.2864	0.3461	0.1187	0.1001
.05	0.1646	0.2399	0.0567	0.0501
.01	0.0209	0.0741	0.0066	0.0100

Power of the test for censored normal with sample 50 censored at 30 (Using CvM) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.8183	0.9999	0.7993	0.9787	0.9642
.15	0.7760	0.9999	0.7579	0.9692	0.9554
.10	0.7036	0.9998	0.6943	0.9527	0.9353
.05	0.5812	0.9992	0.5876	0.9169	0.8979
.01	0.3110	0.9901		0.7640	0.7467

Sign. Level	D.E.	t(3)	Logistic	Normal
.20	0.4922	0.5163	0.2504	0.2001
.15	0.4235	0.4566	0.1934	0.1500
.10	0.3148	0.3733	0.1234	0.1002
.05	0.1887	0.2571	0.0575	0.0502
.01	0.0258	0.0864	0.0052	0.0100

Power of the test for censored normal with sample 55 censored at 33 (Using CvM) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.8473	0.9999	0.8271	0.9870	0.9752
.15	0.8097	0.9999	0.7863	0.9804	0.9660
.10	0.7588	0.9999	0.7391	0.9700	0.9572
.05	0.6331	0.9996	0.6266	0.9399	0.9260
.01	0.3613	0.9959	0.3725	0.8163	0.8007

Sign. Level	D.E.	t(3)	Logistic	Normal
.20	0.5197	0.5432	0.2500	0.2001
.15	0.4452	0.4793	0.1911	0.1501
.10	0.3663	0.4120	0.1382	0.1001
.05	0.2173	0.2837	0.0638	0.0501
.01	0.0376	0.0998	0.0060	0.0100

Power of the test for censored normal with sample 60 censored at 36 (Using CvM) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.8799	1.0000	0.8575	0.9924	0.9801
.15	0.8487	1.0000	0.8263	0.9889	0.9751
.10	0.7944	1.0000	0.7749	0.9804	0.9649
.05	0.6893	0.9998	0.6813	0.9624	0.9422
.01	0.4180	0.9981	0.4373	0.8701	0.8561

Sign. Level	D.E.	t(3)	Logistic	Normal
.20	0.5606	0.5702	0.2686	0.1999
.15	0.4893	0.5134	0.2101	0.1501
.10	0.3951	0.4387	0.1441	0.1001
.05	0.2564	0.3241	0.0707	0.0501
.01	0.0557	0.1255	0.0079	0.0101

Critical Values for the New Suggested Test for Censored Normal at censored ratio 0.6 with Sample Size = 10 (5) 60 (Using AD) (at Significance Levels .2,.15,.1,.05,.01)

N	0.20	0.15	0.10	0.05	.01
10	0.1723	0.1959	0.2290	0.2970	0.5258
15	0.1871	0.2126	0.2527	0.3381	0.6090
20	0.1975	0.2256	0.2648	0.3410	0.6525
25	0.1998	0.2261	0.2655	0.3381	0.5799
30	0.2051	0.2308	0.2705	0.3469	0.5667
35	0.2064	0.2335	0.2718	0.3447	0.5349
40	0.2082	0.2332	0.2719	0.3424	0.5325
45	0.2080	0.2334	0.2712	0.3452	0.5498
50	0.2083	0.2352	0.2730	0.3472	0.5501
55	0.2102	0.2373	0.2720	0.3447	0.5617
60	0.2104	0.2367	0.2719	0.3388	0.5373

Power of Tests for the Censord Normal at Censored ratio 0.6 for Sample Size = 10 (5) 60 (Using AD) (at Significance Levels .2,.15,.1,.05,.01)

N	0.20	0.15	0.10	0.05	.01
10	0.2096	0.1550	0.1067	0.0546	0.0135
15	0.2101	0.1555	0.0998	0.0461	0.0102
20	0.2016	0.1466	0.0989	0.0507	0.0079
25	0.2053	0.1524	0.1013	0.0518	0.0094
30	0.1990	0.1531	0.1015	0.0528	0.0107
35	0.1986	0.1506	0.1063	0.0523	0.0122
40	0.1982	0.1521	0.1003	0.0506	0.0107
45	0.1989	0.1537	0.0995	0.0464	0.0126
50	0.2080	0.1539	0.1040	0.0480	0.0111
55	0.2056	0.1508	0.1036	0.0494	0.0073
60	0.2019	0.1534	0.1046	0.0526	0.0100

Power of the test for censored normal with sample 10 censored at 6 (Using AD) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.3419	0.7118	0.3599	0.4963	0.4059
.15	0.2751	0.6654	0.2949	0.4284	0.3348
.10	0.2050	0.5971	0.2306	0.3528	0.2630
.05	0.1233	0.4684	0.1418	0.2399	0.1706
.01	0.0339	0.2395	0.0421	0.0890	0.0038

Sign. Level	D.E.	t(3)	logistic	NORMAL
.20	0.2122	0.2229	0.2027	0.2000
.15	0.1522	0.1662	0.1455	0.1501
.10	0.0974	0.1092	0.0938	0.1002
.05	0.0359	0.0499	0.0417	0.0501
.01	0.0052	0.0068	0.0080	0.0100

Power of the test for censored normal with sample 15 censored at 9 (Using AD) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.4246	0.8799	0.4261	0.6402	0.5728
.15	0.3538	0.8475	0.3649	0.5795	0.5146
.10	0.2676	0.7951	0.2867	0.5001	0.4448
.05	0.1668	0.6864	0.1843	0.3649	0.3253
.01	0.0522	0.4361	0.0610	0.1545	0.1229

Sign. Level	D.E.	t(3)	logistic	NORMAL
.20	0.2462	0.2749	0.2061	0.2001
.15	0.1831	0.2173	0.1526	0.1501
.10	0.1145	0.1504	0.0948	0.1002
.05	0.0453	0.0747	0.0413	0.0502
.01	0.0033	0.0102	0.0064	0.0101

Power of the test for censored normal with sample 20 censored at 12 (Using AD) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.4978	0.9503	0.4993	0.7479	0.6894
.15	0.4249	0.9317	0.4347	0.6905	0.6424
.10	0.3405	0.9039	0.3562	0.6159	0.5754
.05	0.2369	0.8421	0.2460	0.4957	0.4751
.01	0.0692	0.6067	0.0794	0.2238	0.2198

Sign. Level	D.E.	t(3)	logistic	NORMAL
.20	0.2697	0.3043	0.2109	0.2000
.15	0.2071	0.2442	0.1532	0.1500
.10	0.1416	0.1826	0.0989	0.1001
.05	0.0685	0.1073	0.0482	0.0502
.01	0.0050	0.0160	0.0050	0.0100

Power of the test for censored normal with sample 25 censored at 15 (Using AD) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.5838	0.9829	0.5764	0.8411	0.7809
.15	0.5181	0.9742	0.5128	0.7968	0.7426
.10	0.4315	0.9617	0.4292	0.7352	0.6924
.05	0.3151	0.9304	0.3173	0.6324	0.6109
.01	0.1214	0.7991	0.1359	0.3770	0.3912

Sign. Level	D.E.	t(3)	logistic	NORMAL
.20	0.3187	0.3622	0.2253	0.2001
.15	0.2549	0.3036	0.1681	0.1501
.10	0.1804	0.2338	0.1124	0.1002
.05	0.0973	0.1498	0.0550	0.0502
.01	0.0124	0.0386	0.0081	0.0101

Power of the test for censored normal with sample 30 censored at 18 (Using AD) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.6465	0.9932	0.6281	0.8908	0.8377
.15	0.5807	0.9906	0.5689	0.8599	0.8098
.10	0.4933	0.9850	0.4894	0.8143	0.7702
.05	0.3686	0.9690	0.3709	0.7239	0.6917
.01	0.1675	0.8966	0.1751	0.4899	0.5040

Sign. Level	D.E.	t(3)	logistic	NORMAL
.20	0.3487	0.4012	0.2228	0.2000
.15	0.2866	0.3418	0.1729	0.1500
.10	0.2091	0.2708	0.1149	0.1002
.05	0.1155	0.1817	0.0557	0.0501
.01	0.0195	0.0638	0.0092	0.0100

Power of the test for censored normal with sample 35 censored at 21 (Using AD) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.7134	0.9978	0.6814	0.9321	0.8897
.15	0.6558	0.9966	0.6232	0.9091	0.8673
.10	0.5744	0.9939	0.5482	0.8721	0.8345
.05	0.4384	0.9871	0.4287	0.8007	0.7726
.01	0.2294	0.9541	0.2366	0.6053	0.6219

Sign. Level	D.E.	t(3)	logistic	NORMAL
.20	0.3898	0.4434	0.2358	0.2001
.15	0.3214	0.3810	0.1798	0.1500
.10	0.2460	0.3084	0.1246	0.1002
.05	0.1454	0.2157	0.0630	0.0502
.01	0.0349	0.0915	0.0108	0.0100

Power of the test for censored normal with sample 40 censored at 24 (Using AD) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.7659	0.9994	0.7366	0.9572	0.9253
.15	0.7144	0.9992	0.6808	0.9405	0.9113
.10	0.6404	0.9985	0.6021	0.9100	0.8867
.05	0.5090	0.9956	0.4848	0.8565	0.8390
.01	0.2734	0.9775	0.2748	0.6907	0.7037

Sign. Level	D.E.	t(3)	logistic	NORMAL
.20	0.4333	0.4769	0.2452	0.2000
.15	0.3649	0.4233	0.1944	0.1500
.10	0.2795	0.3516	0.1358	0.1001
.05	0.1772	0.2561	0.0694	0.0501
.01	0.0461	0.1191	0.0132	0.0101

Power of the test for censored normal with sample 45 censored at 27 (Using AD) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.8173	1.0000	0.7751	0.9757	0.9482
.15	0.7659	1.0000	0.7285	0.9652	0.9372
.10	0.6956	0.9997	0.6598	0.9495	0.9163
.05	0.5713	0.9986	0.5447	0.9062	0.8754
.01	0.3051	0.9891	0.3046	0.7500	0.7516

Sign. Level	D.E.	t(3)	logistic	NORMAL
.20	0.4760	0.5141	0.2645	0.2001
.15	0.4054	0.4636	0.2069	0.1501
.10	0.3193	0.3940	0.1420	0.1002
.05	0.2022	0.2919	0.0727	0.0502
.01	0.0498	0.1319	0.0120	0.0100

Power of the test for censored normal with sample 50 censored at 30 (Using AD) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.8512	1.0000	0.8096	0.9856	0.9679
.15	0.8094	1.0000	0.7649	0.9765	0.9579
.10	0.7489	0.9999	0.7032	0.9649	0.9438
.05	0.6226	0.9997	0.5898	0.9311	0.9136
.01	0.3568	0.9954	0.3489	0.8018	0.8145

Sign. Level	D.E.	t(3)	logistic	NORMAL
.20	0.5108	0.5498	0.2760	0.2000
.15	0.4392	0.4917	0.2136	0.1501
.10	0.3506	0.4222	0.1494	0.1001
.05	0.2280	0.3162	0.0749	0.0501
.01	0.0605	0.1471	0.0124	0.0100

Power of the test for censored normal with sample 55 censored at 33 (Using AD) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.8831	1.0000	0.8433	0.9916	0.9768
.15	0.8475	1.0000	0.7979	0.9874	0.9701
.10	0.7981	0.9999	0.7425	0.9785	0.9604
.05	0.6782	0.9998	0.6337	0.9549	0.9374
.01	0.3979	0.9977	0.3818	0.8433	0.8474

Sign. Level	D.E.	t(3)	logistic	NORMAL
.20	0.5378	0.5799	0.2771	0.2001
.15	0.4696	0.5210	0.2197	0.1500
.10	0.3884	0.4584	0.1611	0.1002
.05	0.2572	0.3447	0.0849	0.0502
.01	0.0688	0.1592	0.0130	0.0100

Power of the test for censored normal with sample 60 censored at 36 (Using AD) (Normal against one of the following:)

Sign. Level	Unif.	Chi(1)	Chi(4)	Exp.	Cauchy
.20	0.9067	1.0000	0.8690	0.9946	0.9810
.15	0.8779	1.0000	0.8321	0.9924	0.9768
.10	0.8359	1.0000	0.7854	0.9874	0.9691
.05	0.7374	1.0000	0.6906	0.9731	0.9516
.01	0.4727	0.9991	0.4475	0.8970	0.8886

Sign. Level	D.E.	t(3)	logistic	NORMAL
.20	0.5737	0.6019	0.2925	0.2000
.15	0.5052	0.5481	0.2316	0.1500
.10	0.4220	0.4857	0.1700	0.1002
.05	0.3003	0.3864	0.0955	0.0503
.01	0.0963	0.1934	0.0169	0.0101

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