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On the Local Continuity of a Real Invertible Function

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***Abstract*— In this paper we present a real invertible function which is continuous in a real fixed point for which its inverse is not be continue in a suitable point.**

***Index Terms*- Invertible function, Continuity, Sequence, Convergence**

***2000 AMS Classification:* 26A09, 26A15, 26A30**

I. INTRODUCTION

It is well known that if I and J are two intervals in R and $f: I \to J$ is a invertible and continuus function on I then the inverse function $f^{-1}: J \to I$ is also continue on J (se egg. [1] or [3]). We ask the following natural question: if the function f is invertible and continuous at a given point $x_0 \in I$, then the inverse function f^{-1} is continuous at the corresponding point $y_0 = f(x_0)$?

This is is not true. We will give a real invertible function $f: R \to R,$ continuous in $x_0 \in R$ but the inverse function f^{-1} is not be continuous in $y_0 = f(x_0)$.

II. THE MAIN RESULT

We will use the next lemma:

Lemma 2.1. If $a, b \in R,\ a < b,$ there exist a invertible functions $f: (a,b) \to (a,b]$ and $g: [a,b) \to (a,b).$

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Proof. Firt we assume $a = 0, b = 1$ and we define the function $h: (0,1) \to (0,1]$,

$$h(x) = \begin{cases} \dfrac{1}{n-1}, if\ x = \dfrac{1}{n}, n \geq 2, \\ x, \quad if\ x \in (0,1), x \neq \dfrac{1}{n}, n \geq 2. \end{cases}$$

Let us remark that h is invertible. Now we define the function $f: (a,b) \to (a,b]$,

$$f(x) = h(\frac{x-a}{b-a}),$$

which is also invertible. Similarly we define the function g (see egg. [2]). □

Using this lemma, for every $n \geq 1$, there exist invertible functions $f_n: (n, n+1) \to (n, n+1]$ and

$$g_n: [\frac{1}{n+1}, \frac{1}{n}) \to (\frac{1}{n+1}, \frac{1}{n}).$$

Now we define the function $f: R \to R,$

$$f(x) = \begin{cases} f_n(x), if\ x \in (n, n+1), n \geq 1, \\ g_n(x), if\ x \in [\dfrac{1}{n+1}, \dfrac{1}{n}), n \geq 1, \\ \dfrac{1}{n}, \quad if\ x = n, n \geq 1, \\ x, \quad if\ x \leq 0. \end{cases}$$

Theorem 2.1. The function f defined above is well defined, invertible, continuous in $x_0 = 0$ and the inverse function f^{-1} is not be continuous in $y_0 = f(0).$

Proof. From the way that the above function f was defined it is well define and since the functions f_n and $g_n, n \geq 1,$ are invertible it results that f is invertible. Let us remark that for $x \in R, |x| \le 1$ we have $|f(x)| \le 2|x|,$ so that

$$\lim_{x \to 0} f(x) = 0 = f(0),$$

and then f is continuous in $x = 0.$

Let $y_n = \frac{1}{n} \to 0 = f^{-1}(0).$

Since $f^{-1}(\frac{1}{n}) = n \to \infty$ it results that the inverse

function f^{-1} is not be continuous in $y = 0.$ □

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