A Comparative Analysis of Rate of Convergence For Linear And Quadratic Approximations in N-R Method

Shiv Kumar Sharma

***Abstract* -Newton – Raphson (N-R) Method is commonly used in the solution of algebraic equations and transcendental equations. Using Taylor's theorem for expansion of functions, generally expansion is truncated as linear approximation. In this research work, expansion of function is truncated as quadratic approximation and then a comparative analysis was done for linear and quadratic approximations. Equation $\mathbf{f(x) = 2\ Cos\ x - e^x = 0}$ was investigated and it was solved by using linear and quadratic approximations. On comparison, it was found that rate of convergence is higher in quadratic approximation than linear approximation.**

***Index Terms*- N-R Method, Convergence, Linear, Quadratic, approximation.**

I. INTRODUCTION

Several methods are there to solve algebraic and transcendental equations. We use Bisection method, Regula falsi method, Secant method. Newton Raphson method is far superior to these methods because of higher rate of convergence. Consider a transcendental equation $f(x) = 0$. If x_0 is an approximate root of the equation $f(x) = 0$ and $x_0 + h$ is a better approximation to the root, then $f(x_0 +$ *h=0.* When this equation is expanded by taylor's theorem then we have

$$f(x_0 + h) = f(x_0) + hf'(x_0) + \frac{h^2}{2!}f''(x_0) + \frac{h^3}{3!}f'''(x_0) + \frac{h^4}{4!}f^{iv}(x_0) + \cdots \ldots \ldots \ldots \ldots \ldots = 0$$

We can truncate above expression as a linear function or we can take quadratic approximation. For most of the cases linear approximation is taken as $f(x_0) + hf'(x_0) = 0$. It gives us

$h = -\frac{f(x_0)}{f'(x_0)}$. Therefore first approximation to the root will be $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$. In general we can write $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

If same expansion is truncated as a quadratic approximation as follows: $f(x_0) + hf'(x_0) + \frac{h^2}{2!}f''(x_0) = 0$, which will give us $h = -\frac{f'(x_0) \pm \sqrt{f'(x_0)^2 - 4\frac{f''(x_0)}{2!}}}{2\frac{f''(x_0)}{2!}}$. So,

$x_1 = x_0 - \frac{f'(x_0) \pm \sqrt{f'(x_0)^2 - 4\frac{f''(x_0)}{2!}}}{2\frac{f''(x_0)}{2!}}$. In general, again we can write $x_{n+1} = x_n - \frac{f'(x_n) \pm \sqrt{f'(x_n)^2 - 4\frac{f''(x_n)}{2!}}}{2\frac{f''(x_n)}{2!}}$. Obviously, Quadratic approximation will reduce in smaller number of iteration, while it will take more computation time.

.

II. LITERATURE REVIEW

Ehiwario, J.C., Aghamie, S. have discussed that the Secant method is very more effective in comparison to other methods. Rate of convergence of secant method is very close to Newton – Raphson method and it requires evaluation of only a single function evaluation per iteration. They have also discussed that rate of convergence of bisection method is slow.

Waqas Nazeer at al., have suggested and analyzed two new iterative methods for solving nonlinear scalar equations namely: the modified generalized Newton Raphson's method and generalized Newton Raphson's method. These methods are free from second derivative and have order of convergence 6 and 5 respectively.

Tan Tingting at al, have applied the N - R method with basis of current injection into a case of distributional network. First of all, the corrector equations for these 2 methods were derived and then compared. The Jacobian (J) matrix of the conventional N-R method are recalculated in each iteration, On the other hand, this Newton-Raphson method based on current injection only needs re-calculation of the diagonal elements of its Jacobian matrix which mainly consists of admittance matrix's elements.

Saba Akram and Qurrat ul Ann have concluded in their research that rate of convergence of Newton Raphson method is faster than other methods, which can make the computer programming better and will reduce computation time. They have also discussed that the rate of convergence of bisection method is very low therefore it's difficult to apply such kind of systems in equations.

Azizul Hasan has discussed comparison of secant method with newton Raphson, bisection and regula falsi method.

Muhammad Saqib et. al have presented Solution of non linear equations having cubic convergence by using newton Raphson method and they have shown polynomio-graphy. Polynomio-graphy is very useful in its applications in the field of arts and science. In this research, they have obtained polynomio-graphs for different types of complex polynomials. The polynomiographs so obtained were quite new and interesting. Researcher strongly believes that the results of this research work have enriched software of existing polynomiography.

Shiv Kumar Sharma Assistant Professor, Amity University Jaipur

III. CALCULATION

We have investigated a transcendental equation $f(x) = 2\,Cos\ x - e^x = 0$ using linear approximations and quadratic approximations.

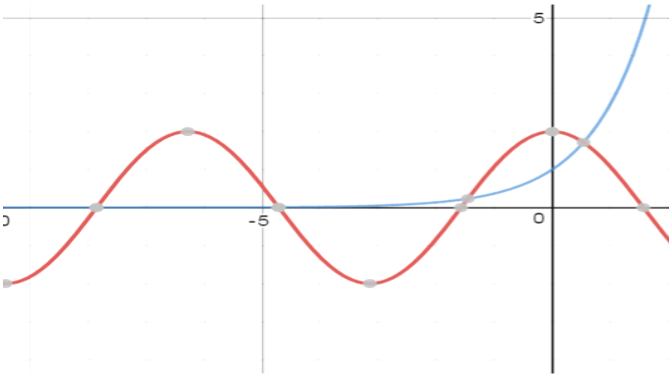


Figure 1 : Solution Set of 2 Cos x - exp.x

Now Calculate value of function at points $x = 0$ *and* $x = 1$ as follows:

Value of Function $f(x) = 2\,Cos\ x - e^x$	Point
1	$x = 0$
- 1.637677216	$x = 1$

Since, function changes its value from positive to negative between the points $x = 0$ *and* $x = 1$. Therefore a root of equation lies between the points $,x = 0$ *and* $x = 1$. Moreover, $x = 0$ is closer to the actual root. So, taking Initial approximation as, $x_0 = 0$. We have results of linear approximation and quadratic approximation as follows:

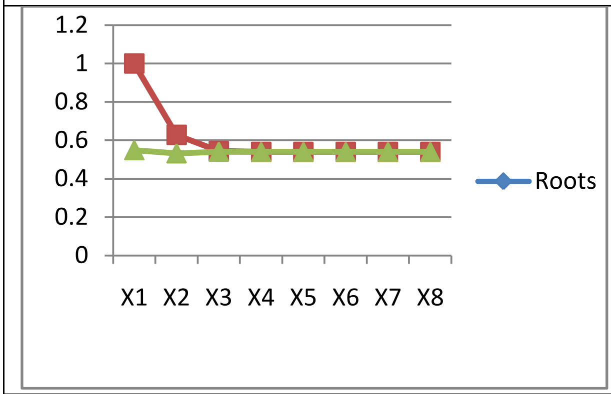
TABLE -1: COMPARATIVE ROOTS IN SUCCESSIVE ITERATIONS

	Linear Approximation		Quadratic Approximation	
	Root	Error %	Root	Error %
x_1	1	85.25889078%	0.548583770	1.6299095740%
x_2	0.627904125	16.32482171%	0.532279965	-1.390404101%
x_3	0.544206631	0.819116813%	0.540698363	0.169178974%
x_4	0.539797325	0.002253674%	0.539785160	0.000000000%
x_5	0.539785160	0.000000000%	0.539785160	0.000000000%
x_6	0.539785161	0.000000000%	0.539785160	0.000000000%
x_7	0.539785160	0.000000000%	0.539785160	0.000000000%
x_8	0.539785161	0.000000000%	0.539785160	0.000000000%

IV. UNITS

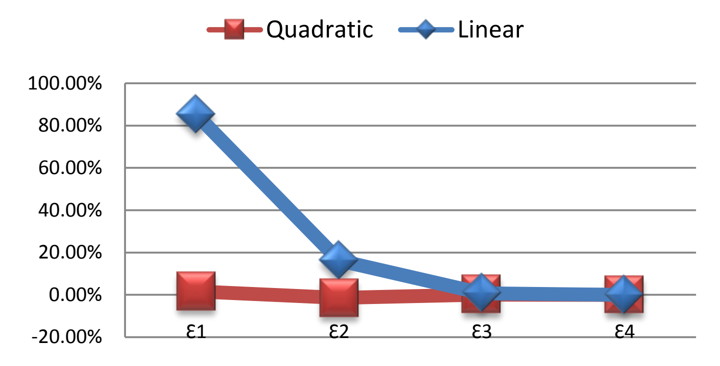
we can see in table that in quadratic approximation optimal root is obtained after 4th iteration while that in linear approximation optimal root is obtained after 5th iteration.

FIGURE- 1: COMPARATIVE RATE OF CONVERGENCE



From above chart it is clear that quadratic approximation is converging more rapidly than linear approximation in Newton – Raphson method.

Figure-2: Comparative Errors



Moreover, magnitude of error (16.325%) in linear approximation is much higher than the error (1.39%) in quadratic approximation after second iteration.

V. CONCLUSION

We obtain best possible root after 5th iteration in linear approximation and after 4th iteration in quadratic approximation. So, we can conclude that rate of convergence in case of quadratic approximation is higher, but calculations are more time consuming. Therefore, on analysis, it was found that linear approximation can be regarded as superior method. This is the reason, due to which we do not go to quadratic approximation, cubic approximation or truncation at higher degree terms.

REFERENCES

[1] Ehiwario, J.C., Aghamie, S.O (2014), Comparative Study of Bisection, Newton-Raphson and Secant Methods of Root- Finding Problems, IOSR Journal of Engineering (IOSRJEN) www.iosrjen.org ISSN (e): 2250-3021, ISSN (p): 2278-8719 Vol. 04, Issue 04 (April. 2014), ||V1|| PP 01-07

[2] Tan Tingting, Li Yang, Li Ying, and Jiang Tong (2013), The Analysis of the Convergence of Newton-Raphson Method Based on the Current Injection in Distribution

Network Case, International Journal of Computer and Electrical Engineering, Vol. 5, No. 3, pp 288-290

[3] Saba Akram, Qurrat ul Ann (2015), Newton Raphson Method, International Journal of Scientific & Engineering Research, Volume 6, Issue 7, July-2015 1748 ISSN 2229-5518 pp 1748-1752

[4] Waqas Nazeer, Amir Naseem, Shin Min Kang, Young Chel Kwun,(2016) Generalized Newton Raphson's method free fromsecond derivative, J. Nonlinear Sci. Appl. 9 (2016), pp 2823-2831

[5] Biswa Nath Datta (2012), Lecture Notes on Numerical Solution of root Finding Problems. Www.math.niu.edu/dattab.

[6] Azizul Hasan (2016), Numerical Study of Some Iterative Methods for Solving Nonlinear Equations, International Journal of Engineering Science Invention ISSN (Online): 2319 – 6734, ISSN (Print): 2319 – 6726 www.ijesi.org ||Volume 5 Issue 2|| February 2016 || PP.01-10

[7] S. M. Kang, W. Nazeer, M. Tanveer, Q. Mehmood, K. Rehman (2015) Improvements in Newton-Rapshon method for nonlinear equations using modified Adomian decomposition method, Int. J. Math. Anal., 9 pp 1919–1928.

[8] J. Kou, The improvement of modified Newton's method, Appl. Math. Comput., 189 (2007), pp 602–609

[9] S. Weerakoon, T. G. I. Fernando (2000) A variant of Newton's method with accelerated third-order convergence, Appl. Math. Lett., 13 pp 87–93

[10] Muhammad Saqib, Adnan Ashgar, Amir Naseem (2016), dynamics of improved newton-raphson method, Sci.Int.(Lahore),28(5), ISSN: 1013-5316; pp 4621-4626

Shiv Kumar Sharma is B.E.(Mechanical Engineering), MBA (HR) and M.Tech. (Thermal System& Design). Presently he is working as Assistant Professor in the department of Mechanical and Automation Engineering in Amity University Jaipur