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Geo/Geo/1/N Queue with Retention of Reneging Customers

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***Abstract*—This paper considers a Geo/Geo/1/N queue, in which customers may renege and reneging customers may be retained. The steady state distribution of the number of customers in the system is derived.**

***Index Terms*—Discrete-time queue, geometric service, reneging, retention**

I. Introduction

The phenomenon of customer reneging is commonly observed in queueing systems, customers may leave a service system before receiving service due to the long waiting time. The problem of queues with customer reneging was first analyzed by Palm [1]. A bibliography can be found in Gross et al. [2]. Kumar and Sharma [3], [4] envisaged that reneging customers may be retained for his future service owing to a certain customer retention strategy and studied the retention of reneging customers.Kumar and Sharma [5] studied a finite capacity multi-server Markovian queueing system with discouraged arrivals, reneging and retention of reneging customers. They considered homogeneous servers. Kumar and Sharma [6] studied an M/M/c/N queueing model with reneging and retention of reneging customers. Kumar [7] obtained the transient solution of an M/M/c/N queueing model with balking, reneging, and retention of reneging customers.Kumar and Sharma [8] considered a finite capacity Markovian queueing system with two heterogeneous servers, discouraged arrivals, reneging, and retention of reneging customers.

This paper considers a discrete-time Geo/Geo/1/N queue with retention of reneging customers. The steady state distribution of the number of customers in the system is derived.

The paper is organized as follows. In Section II, we described the queueing model. In Section III, we formulate the system as a one-dimensional discrete-time Markov chain and find the stationary probability vector of the number of customers in the system. Conclusion is provided in Section IV.

II. Model

In this section, we formulate the queueing model, which is based on the following assumptions:

1. We consider a discrete-time queueing system in which the time axis is divided into fixed-length contiguous intervals, referred to as slots.
2. Customers arrive according to a geometric process. Let $p$be the probability that a customer enters the system during a slot.
3. The service of a customer can start only at a slot boundary.
4. The service times of customers follow a geometric distribution with parameter q.
5. The system has a buffer of finitecapacityN.
6. Customers are served in FCFS order.
7. The reneging times follows a geometric distribution with parameter r.
8. Reneging customers may leave the queue without getting service with probability s.

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III. Stationary Distribution

To model this system, we define a discrete-time Markov chain:

$$\{N_k, k \geq 0\} \tag{1}$$

whereN_k denotes the total number of customers in the system at the end of slot k. The state space of this Markov chain is:

$$\{0, 1, 2, \ldots, N\} \tag{2}$$

The one-step transition probability matrix $\boldsymbol{P}$ is given by:

$$\boldsymbol{P} = \boldsymbol{P_0}.*\boldsymbol{r_0} + \boldsymbol{P_1}.*\boldsymbol{r_1} + \boldsymbol{P_2}.*\boldsymbol{r_2} + \cdots + \boldsymbol{P_N}.*\boldsymbol{r_N} \tag{3}$$

where $\boldsymbol{P_0}$ isan $(N+1) \times (N+1)$ matrix with its ijelements$(\boldsymbol{P_0})_{i,j}$:

$$(\boldsymbol{P_0})_{0,0} = 1 - p \tag{4}$$

$$(\boldsymbol{P_0})_{0,1} = p \tag{5}$$

$$(\boldsymbol{P_0})_{i,i-1} = (1-p)q, \quad 1 \leq i \leq N \tag{6}$$

$$(\boldsymbol{P_0})_{i,i} = pq + (1-p)(1-q), \quad 1 \leq i \leq N-1 \tag{7}$$

$$(\boldsymbol{P_0})_{i,i+1} = p(1-q), \quad 1 \leq i \leq N-1 \tag{8}$$

$$(\boldsymbol{P_0})_{N,N} = 1 - p(1-q) \tag{9}$$

The matrix $\boldsymbol{P_i}$isan $(N+1) \times (N+1)$ matrix obtained by deleting first i columns from matrix $\boldsymbol{P_0}$ and add $i$zerocolumns, whose elements are all zero, at the end of matrix $\boldsymbol{P_0}$ .The matrix $\boldsymbol{r_i}$ is an $(N+1) \times (N+1)$ matrix given by:

$$\boldsymbol{r_0} = \begin{pmatrix} 1 & \cdots & 1 \\ 1 & \cdots & 1 \\ r_{1,0} & \cdots & r_{1,0} \\ \vdots & \ddots & \vdots \\ r_{N-1,0} & \cdots & r_{N-1,0} \end{pmatrix} \quad (10)$$

$$\boldsymbol{r_i} = \begin{pmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ r_{1,i} & \cdots & r_{1,i} \\ \vdots & \ddots & \vdots \\ r_{N-1,i} & \cdots & r_{N-1,i} \end{pmatrix}, \ 1 \le i \le N \quad (11)$$

where$r_{i,j}$ is the probability that j customers among i waiting customers leave the system due to reneging at a slot: for $i \ge j$,

$$r_{i,j} = \sum_{k=j}^{i} \binom{i}{k} r^k (1-r)^{i-k} \binom{k}{j} s^j (1-s)^{k-j} \quad (12)$$

for $i < j$, the value of $r_{i,j}$ is 0. The operator $.*$ in Eq. (3) denotes the element-wise multiplication.

Let $\boldsymbol{x}$ be the stationary probability vector associated with the discrete-time Markov chain $\{N_k, k \ge 0\}$:

$$\boldsymbol{x} \equiv (x_0, x_1, \dots, x_N) \quad (13)$$

where:

$$x_i \equiv \lim_{k\to\infty} P\{N_k = i\} \quad (14)$$

Then, the stationary probability $\boldsymbol{x}$ is obtained by solving

$$\boldsymbol{xP} = \boldsymbol{x} \quad (13)$$

$$\boldsymbol{xe} = 1(14)$$

where$\boldsymbol{e}$is the column vector with all element 1.

IV. Conclusion

In this paper, we have considered a Geo/Geo/1/N queue with impatient customers, where customers may renege and reneging customers may be retained. The steady state distribution of the number of customers in the system has been derived.

References

[1] C. Palm, Methods of judging the annoyance caused by congestion, *Telecommunications*, 4, 1953, 153-163.

[2] D. Gross, J. F. Shortle, J. M. Thompson, and C. M. Harris, *Fundamentals of Queueing Theory*, John Wiley & Sons, 4th Ed., 2008.

[3] R. Kumar and S. K. Sharma, M/M/1/N queueing system with retention of reneged customers, *Pakistan Journal of Statistics and Operation Research*, 8(4), 2012, 859-866.

[4] R. Kumar and S. K. Sharma, An M/M/1/N queueing system with retention of reneged customers and balking, *American Journal of Operational Research*, 2(1), 2012, 1-5.

[5] R. Kumar and S. K. Sharma, A multi-server Markovian queueing system with discouraged arrivals and retention of reneged customers, *International Journal of Operational Research*, 9(4), 2012, 173-184.

[6] R. Kumar and S. K. Sharma, An M/M/c/N queueing system with reneging and retention of reneged customers, *International Journal of Operational Research*, 17(3), 2013, 333-334.

[7] R. Kumar, Economic analysis of an M/M/c/N queueing model with balking, reneging and retention of reneged customers, *Opsearch*, 50(3), 2013, 383-403.

[8] R. Kumar and S. K. Sharma, Two-heterogeneous server Markovian queueing system with discouraged arrivals, reneging and retention of reneged customers, *International Journal of Operational Research*, 11(2), 2014, 64-68.