World Journal of Research and Review (WJRR)
ISSN:2455-3956, Volume-4, Issue-2, February 2017 Pages 18-19

Performability of Queue with Interrupted Repairs

Yutae Lee

***Abstract*—We consider a queueing system, consisting of a Poisson input stream and a single server. The server is subject to breakdowns. The failed server requires repair at a facility, which has an unreliable repair crew. The repair crew also subjects to breakdown when it is repairing. We obtain the availability and the delay distribution of the queueing system.**

***Index Terms*—Availability, breakdown, delay, interrupted repair**

I. Introduction

Perfectly reliable servers are virtually nonexistent in many practical situations. For this reason, great attention has been paid to queueing systems with serversbreakdowns. Aissani[1], Choudhury and Ke[2], Falin [3], Gharbi et al. [4], Lee [5], and others considered the unreliable queues wherein customers who find the server broken down should wait in the queue until the server is repaired. The failed server requires repair at a facility, which has a repair crew.

In realistic environments the repair crew is possible to break down when it is repairing. Therefore, considering an unreliable repair crew in a repairable system is also practical and imperative [6]. Lee [7]considered a queue with breakdowns and interrupted repairs. Within the framework of M/M/1 queue, the steady-state distribution of the system state and the number of customers is derived analytically by using probability generating function method. This paper derives the availability and the delay distribution of the queuing system in [7].

II. Model

A single server queue is considered. Customers arrive according to a Poisson process with rate λ. Service times of customers are independent of each other and have a common exponential distribution with rate μ.The server is assumed to be subject to breakdowns. Failed server is repaired by a repair crew. Once the server is repaired, it is as good as new. The repair crew may function wrongly or fail when it is repairing. It can also be fixed. Once the repair crew is fixed, it can function again. It is assumed that all relevant random variables, such as the times to failure of the server, the repair times of the failed server, the times to failure of the repair crew, and the repair times of the failed repair crew, have exponential distributions with rates α, β, γ, and δ, respectively. We make all the necessary independence assumptions on the inter-arrival times, the service times, the times to failure of the server, the repair times of the failed server, the times to failure of the repair crew, and the repair times of the failed repair crew.

Let $S(t)$ be the state of the server and the repair crew at time t:$S(t) = 1$ if the server is available at time t; $S(t) = 2$ if the repair crew is repairing the failed server at time t; $S(t) = 3$ if both the server and the repair crew are failed at time t. Define $N(t)$ as the number of customers in system at time t. Then, the process $\{(S(t), N(t))\}$ is a continuous time Markov chain with state space$\{(s, n), s = 1,2,3, n \geq 0\}$.

III. Service Availability

The stochastic process $\{S(t), t \geq 0\}$ is a continuous time Markov chain with state space $\{1,2,3\}$ and infinitesimal generator:

$$\begin{pmatrix} -\alpha & \alpha & 0 \\ \beta & -(\beta+\gamma) & \gamma \\ \beta & 0 & -\beta \end{pmatrix} (1)$$

Letting $\pi_i \equiv \lim_{t\to\infty} P\{S(t) = i\}$, we obtain:

$$\pi_1 = \frac{\beta}{\alpha+\beta} (2)$$

$$\pi_2 = \frac{\alpha}{\alpha+\beta}\frac{\beta}{\beta+\gamma} (3)$$

$$\pi_3 = \frac{\beta}{\alpha+\beta}\frac{\gamma}{\beta+\gamma} (4)$$

Service availability $P_{available}$ is defined as the probability that the service is available regardless of server breakdowns. Hence,

$$P_{available} \equiv \lim_{t\to\infty} P\{S(t) = 1 \text{ or } S(t) = 2\} = \frac{\beta(\alpha+\beta+\gamma)}{(\alpha+\beta)(\beta+\gamma)} \quad (5)$$

IV. Delay Distribution

It is assumed that the system reaches a stationary state.Let $p_{i,n} \equiv \lim_{t\to\infty} P\{S(t) = i, N(t) = n\}$. Define:

$$P_i(z) \equiv \sum_{n=0}^{\infty} p_{i,n} z^n \qquad (6)$$

which is given in [7].

Let Ddenote the delay of an arbitrary customer, assuming that the system has attained an equilibrium state. If:

$$D^*(s) \equiv E(e^{-sD}) (7)$$

This work was supported by the ICT R&D program of MSIP/IITP. [R0101-15-0070, Development of The High Availability Network Operating System for Supporting Non-Stop Active Routing].

Yutae Lee, Department of Information and Communications Engineering, Dongeui University, Busan, Republic of Korea.

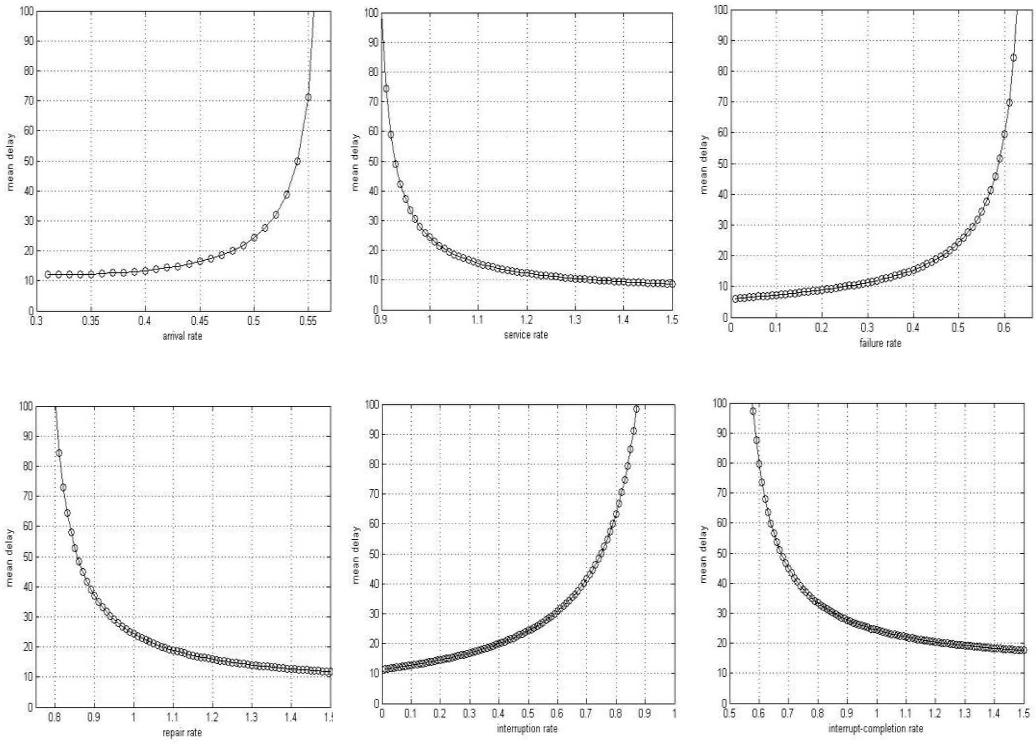

Figure 1. Mean delay

is the Laplace-Steiljes transform of the delay distribution, then:

$$P_1(z) + P_2(z) + P_3(z) = D^*(\lambda(1 - z)) \qquad (8)$$

holds. Therefore, we have:

$$D^*(s) = P_1\left(1 - \frac{s}{\lambda}\right) + P_2\left(1 - \frac{s}{\lambda}\right) + P_3\left(1 - \frac{s}{\lambda}\right) \; (9)$$

V. Numerical Examples

We provide 6 cases for illustration purposes andshow the effects of different system parameterson the steady-state performance:

Case (a):$\lambda \in [0.31, 0.57], \ \ \mu = 1$
$\alpha = 0.5, \ \ \beta = 1, \ \ \gamma = 0.5, \ \ \delta = 1$
Case (b): $\lambda = 0.5, \ \ \mu \in [0.90, 1.50]$
$\alpha = 0.5, \ \ \beta = 1, \ \ \gamma = 0.5, \ \ \delta = 1$
Case (c): $\lambda = 0.5, \ \ \mu = 1$
$\alpha \in [0.01, 0.66], \ \ \beta = 1, \ \ \gamma = 0.5, \ \ \delta = 1$
Case (d): $\lambda = 0.5, \ \ \mu = 1$
$\alpha = 0.5, \ \ \beta \in [0.76, 1.50], \ \ \gamma = 0.5, \ \ \delta = 1$
Case (e): $\lambda = 0.5, \ \ \mu = 1$
$\alpha = 0.5, \ \ \beta = 1, \ \ \gamma \in [0.01, 0.99], \ \ \delta = 1$
Case (f): $\lambda = 0.5, \ \ \mu = 1$
$\alpha = 0.5, \ \ \beta = 1, \ \ \gamma = 0.5, \ \ \delta \in [0.51, 1.50]$

The effects of various parameters on the mean number of customers in the system are shown in Figure 1.

VI. Conclusion

We considered a queueing system, consisting of a Poisson input stream and a server. The server is subject to breakdowns. The failed server requires repair at a facility, which has an unreliable repair crew. The repair crew also subjects to breakdown when it is repairing. This paper obtained the steady-state performance of the queueing system with server breakdowns and interrupted repairs.

References

[1] A. Aissani, A retrial queue with redundancy and unreliable server, *Queueing Systems*, 17(3-4), 1994, 431-449.

[2] G. Choudhury and J.-C. Ke, An unreliable retrial queue with delaying repair and general retrial times under Bernoullivacation schedule, *Applied Mathematics and Computation*, 230, 2014, 436-450.

[3] G. Falin, An M/G/1 retrial queue with an unreliable server and general repair times, *Performance Evaluation*, 67(7), 2010, 569-582.

[4] N. Gharbi, B. Nemmouchi, L. Mokdad, and J. Ben-Othman, The impact of breakdowns disciplines and repeated attemptson performance of small cell networks, *J. Computational Science*, 5(4), 2014, 633-644.

[5] Y. Lee, Discrete-time bulk-service queue with Markovian service interruption and probabilistic bulk size, *J. Appl. Math. & Informatics*, 28(1-2), 2010, 275-282.

[6] C.-C. Kuo and J.-C. Ke, Comparative analysis of standby systems with unreliable server and switching failure, *ReliabilityEngineering and System Safety*, 145, 2016, 74-82.

[7] Y. Lee, Queue with breakdowns and interrupted repairs, *The International Journal of Engineering and Science*,accepted, 2017.