World Journal of Research and Review (WJRR)
ISSN:2455-3956, Volume-4, Issue-2, February 2017 Pages 16-17

Delay of $\mathrm{Geo}^X/\mathrm{Geo}/1$ N-limited Nonstop Forwarding Queue

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***Abstract*—Nonstop forwarding is designed to minimize packet loss when a management system fails to function. A system with non-stop forwarding can continue to forward some packets even in the event of a processor failure. We consider a $\mathrm{Geo}^X/\mathrm{Geo}/1$ N-limited nonstop forwarding queue. In this queueing system, when the server breaks down, up to N customers can be serviced during the repair time.The delay distribution of customers is given by matrix geometric analysis.**

***Index Terms*—Delay, discrete-time queue, geometric service, N-limited nonstop forwarding**

I. Introduction

Queueing systems with servers subject to breakdowns and repairs have been studied extensively [1, 2]. Recently, Falin [3], Lee [4], Choudhury and Ke [5], and others considered the unreliable queues wherein, when a server fails, customers in the system should wait for the server to be repaired without being served. However, there are other practical situations: when a server breakdown occurs, the system continues to forward some customers without stopping immediately. For example, nonstop forwardingis designed to minimize packet loss during a management system failure by maintaining L2 and L3 forwarding, respectively, where the system continues forwarding some packets in the event of a processor failure [6, 7].

Motivated by this factor, we consider a discrete-time N-limited nonstop forwarding, where the service continues instead of being stopped completely even in the case that the server is defective. The server starts immediately the repair process whenever the server breaks down. Despite the server breakdown, up to N customers can be serviced during the repair time.Lee [8]analyzed a discrete-time N-limited nonstop forwarding queue with batch geometric arrivals and deterministic service times. In [9] and [10], Lee and Choi analyzed the steady-state distribution of the number of customers in Geo/Geo/1 queue and $\mathrm{Geo}^X/\mathrm{Geo}/1$ queue, respectively, with N-limited nonstop forwarding. In this paper, we analyze the delay distribution of customers of $\mathrm{Geo}^X/\mathrm{Geo}/1$N-limited nonstop forwarding queue.

II. Model

This paper considers a Geo^X / Geo / 1 queue in which the time axis is divided into fixed-length contiguous intervals, referred to as slots. Customers arrive according to a batch geometric process. The numbers of arrivals during the consecutive slots are assumed to be independent and identically distributed random variables with distribution$\{a_k, k = 0, 1, \dots\}$. It is assumed that the service of a customer can start only at a slot boundary. The service times of customers are assumed to follow geometric distribution with parameter$q$. The system has a buffer of infinite capacity. Customers are served in FCFS order. The exact location of arrival instants within the slot length are not specified here. It is even irrelevant as long as the system is observed at slot boundaries only.

We assume that the server is subject to breakdowns. The server broken down starts immediately the repair process. The lifetime of the server is assumed to be geometrically distributed with parameterα, where $1-\alpha$ is the probability that a failure does not occur in a slot. The repair times of servers follows a geometrical distribution with parameter β, where $1-\beta$ is the probability that a failure will not be concluded in a slot. When the server breaks down, the system continues to forward the next N customers. The inter-arrival times, the service times, the failure times, and the repair times are assumed to be mutually independent of each other.

Let $M(k)$ be the number of customers in the system at the beginning of slot k. Let $S(k)$ be the server state at the beginning of slot k: the value of $S(k)$ is n if the server is under repair and the system has forwarded $\mathrm{N}-\mathrm{n}$ customers after the server's breakdown; the value of $S(k)$ is $N+1$ if the server is normal.Then $\{(M(k), S(k)), k = 0, 1, \dots\}$ is a Markovian process with state space $\{0, 1, 2\} \times \{0, 1, \dots, N+1\}$.

III. Delay Distribution

Under the equilibrium condition [10, 11]:

$$\sum_{i=1}^{\infty} i a_i < q\left[1-\frac{\alpha}{\alpha+\beta}\left\{\frac{q(1-\beta)}{1-(1-q)(1-\beta)}\right\}^N\right] \tag{1}$$

we can compute the steady state distribution [10]:

$$x_{i,j} \equiv \lim_{k\to\infty} \mathrm{P}\{M(k) = i, S(k) = j\} \tag{2}$$

of the Markov chain.

The delay is defined as the total time period a tagged customer spends in the system, i.e., the number of slots between the end of the customer's arrival slot and the end of its departure slot. Let $d_{i,j}$ be the mean remaining delay of a tagged customer at the beginning of a slot when the number of customers that will be served before the tagged customer is $i-1$ and the system state is j for $i = 1,2,\dots,$ and $j = 0,1,2,\dots,N+1$. Letting:

$$\boldsymbol{d}_i \equiv \left(d_{i,0}, d_{i,1}, \dots, d_{i,N+1}\right)^{\boldsymbol{t}} \tag{3}$$

This work was supported by the ICT R&D program of MSIP/IITP. [R0101-15-0070, Development of The High Availability Network Operating System for Supporting Non-Stop Active Routing].

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$$\boldsymbol{A} \equiv \begin{pmatrix} 1-\beta & 0 & 0 & \cdots & 0 & 0 & \beta \\ 0 & (1-\beta)(1-q) & 0 & \cdots & 0 & 0 & \beta(1-q) \\ 0 & 0 & (1-\beta)(1-q) & \cdots & 0 & 0 & \beta(1-q) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & (1-\beta)(1-q) & 0 & \beta(1-q) \\ 0 & 0 & 0 & \cdots & 0 & (1-\beta)(1-q) & \beta(1-q) \\ 0 & 0 & 0 & \cdots & 0 & \alpha(1-q) & (1-\alpha)(1-q) \end{pmatrix} \tag{4}$$

$$\boldsymbol{B} \equiv \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1-\beta & 0 & 0 & \cdots & 0 & 0 & \beta \\ 0 & 1-\beta & 0 & \cdots & 0 & 0 & \beta \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \beta \\ 0 & 0 & 0 & \cdots & 1-\beta & 0 & \beta \\ 0 & 0 & 0 & \cdots & 0 & \alpha & 1-\alpha \end{pmatrix} \tag{5}$$

we obtain:

$$\boldsymbol{d}_1 = \boldsymbol{A}\boldsymbol{d}_1 + \boldsymbol{e} \tag{6}$$

$$\boldsymbol{d}_n = q\boldsymbol{B}\boldsymbol{d}_{n-1} + \boldsymbol{A}\boldsymbol{d}_n + \boldsymbol{e}, \ \ n \geq 2 \tag{7}$$

Hence:

$$\boldsymbol{d}_n = \sum_{i=0}^{n-1} [(\boldsymbol{I}-\boldsymbol{A})^{-1} q\boldsymbol{B}]^i \, (\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{e}, \ \ n \geq 1 \tag{8}$$

Using the stationary probability vector presented above, we now determine the mean delay $\mathrm{E}(D)$. The mean delay $\mathrm{E}(D)$ of a tagged customer is obtained as in Eq. (9), where $b(k)$ is the probability of a tagged customer being in the kth position of its batch, which is given by:

$$b(k) = \frac{\sum_{j=k}^{\infty} a_j}{\sum_{i=1}^{\infty} i a_i} \tag{10}$$

IV. Conclusion

Nonstop forwarding is designed to minimize packet loss when a management system fails to function. In the nonstop forwarding mechanism, the system can continue to forward some packets in the event of a processor failure. This paper considered a $\mathrm{Geo}^X/\mathrm{Geo}/1$ N-limited nonstop forwarding queue. This paper gave the delay distribution of customers by matrix geometric analysis.

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$$\mathrm{E}(D) = \sum_{k=1}^{\infty} b(k) \left[\sum_{j=0}^{N} x_{0,j}\{\beta d_{k,N+1} + (1-\beta) d_{k,j}\} + x_{0,N+1}\{\alpha d_{k,N} + (1-\alpha) d_{k,N+1}\} \right. \tag{9}$$
$$\left. + \sum_{i=1}^{\infty} \left[\begin{array}{c} x_{i,0}\{\beta d_{i+k,N+1} + (1-\beta) d_{i+k,0}\} \\ + x_{i,N+1}\{\alpha q d_{i+k-1,N} + (1-\alpha) q d_{i+k-1,N+1} + \alpha(1-q) d_{i+k,N} + (1-\alpha)(1-q) d_{i+k,N+1}\} \\ + \sum_{j=1}^{N} x_{i,j}\{\beta q d_{i+k-1,N+1} + (1-\beta) q d_{i+k-1,j-1} + \alpha(1-q) d_{i+k,N+1} + (1-\alpha)(1-q) d_{i+k,j}\} \end{array} \right] \right]$$