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The Role of the Characteristic Load Impedance in Passive Ladder Networks: Again the Presence of the Fibonacci Sequence

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***Abstract*— This paper deals with an analysis of the different behaviors existing between ladder networks (LN) terminated and not terminated by the characteristic impedance Z0. The main purpose of this work is to find, in both cases, links with the golden ratio Φ and, as a consequence, with Fibonacci numbers. Results of some simulations related to the determination of impedances, node voltages and branch currents are given in order to underline approximation effects on the node voltages and branch currents related to both the presence and absence of Z0.This study has been firstly applied to R-R ladder networks but it is here extended to consider other kinds of LNs, having other types of single cells such as C-C ; L-L whose characteristic impedances have also been determined.**

Double resistive LN have also been investigated for which impedances in any cell, currents in any branch and voltages in any node have been determined in the two cases of presence and absence of the characteristic impedance and with different ratio between longitudinal and transversal impedances. This study has another not less relevant aim: find one of the possible platform for the modelization of biosystems like DNA and RNA, because these structures even if much more complex look like a LN

***Index Terms*—Passive ladder Network, DNA,RNA .**

I. INTRODUCTION

Passive ladder networks made by a number of single cells having both longitudinal and transversal impedances have been studied for a long time [1-6] due to their versatility in representing a good model for mechanical, chemical, thermal and electronic systems and also because they were frequently employed in passive filters [7,8]. Their interest to day is still alive due to the new implications in analog neural networks and in new promising devices such as multi gate MOSFET for multi-sensing applications, where a possible model for both the inversion and depletion regions falls into these ladder networks. Furthermore, we cannot exclude in a near future their possible implications in the study of both the electric behavior of both DNA and RNA structures[9], and related aspects of epigenomics. This paper, on the other hand, considers these kinds of networks from another viewpoint: the search of the presence of links with Fibonacci numbers. In many cases these famous numbers are present as expression of impedances, voltages, currents and may facilitate the rapid calculation of their amplitudes[4]. This paper in particular compares,as a new investigation, resistive LNs with and without their characteristic impedance. This last, used as termination of the finite LN, allows the R-R finite LN to be treated as a ladder network having an infinite number of single cells.These considerations are here extensively applied to R-R LNs, and to a less extent to both C-C, L-L LNs.

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II. DETERMINATION OF THE CHARACTERISTIC IMPEDANCES IN A FEW TYPES OF LADDER NETWORKS.

Let us first determine thecharacteristic impedance of a LN formed by a number N of identical cells directly coupled each other and characterized by a longitudinal impedance kZ1 and a transversal impedance lZ2, as shown in fig.l,where k and lare positive and real numbers.

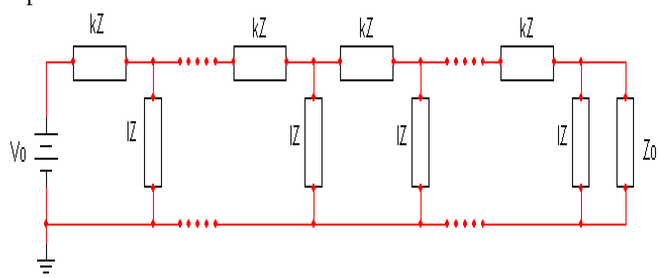


Fig.l - Ladder network with impedances kZ and LZ

We introduce here a definition of the characteristic impedance Z0 useful for our investigation.

A simple remark suggests that the input impedance of the infinite LN cannot be changed if we extract a single cell: then we assume that the input impedance and the matching terminated impedance are identical.

To this first purpose, let us consider first a single cell, adapted by Z_0, as in Fig. 2-a),

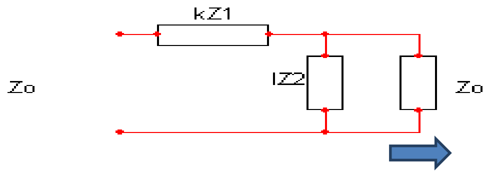


Fig. 2-a) Single cell of a ladder network terminated by the characteristic impedance Z_0.

Therefore,$Z_0$is that impedance value satisfying the following equation:

$$Z_0 = kZ_1 + lZ_2 \parallel Z_0 \quad (1)$$

whichgives:

$$Z_0{}^2 - kZ_1Z_0 - klZ_1Z_2 = 0 \quad (2)$$

whose two solutions are expressed by:

$$Zo = kZ_1 \frac{\left[1 \pm \sqrt{1+4\frac{lZ_2}{kZ_1}}\right]}{2} \quad (3)$$

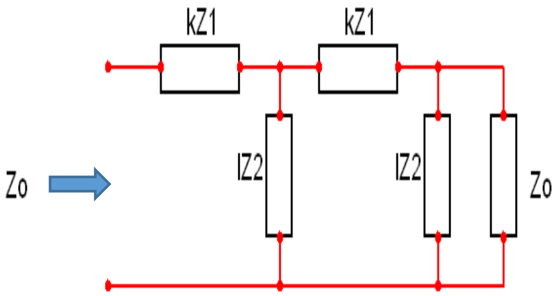


Fig. 2-b) Two cells of a ladder network terminated by the characteristic impedanceZ_0.

It is easy to prove the consistency of the definition expressed by (1). In fact, the second order equation (2) is also obtained from fig.2-b) and also from LNs (fig. l) formed by any number of cells. This result simplifies our considerations: the presence of Z_0 allow us to deal with a discrete and infinite transmission line.

With reference to the relationship (3), Z_0 will always have two solutions, no matter how many cells are considered as it is easy to prove! One of these solutions is positive and will be the only one taken into account in the following.

As a remark, we notice that, if we apply the same procedure to a continuous transmission line represented as a single cell LN shown in fig. 3, we obtain:

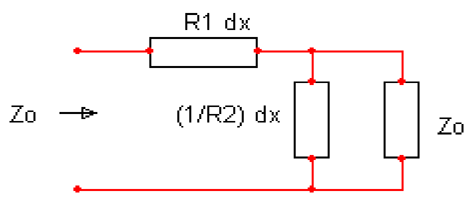


Fig.3 Single cell for a continuous transmission line made by infinitesimal longitudinal resistor and transversal conductance

$$Z_0 = R1 \cdot dx + \frac{Z_0 R_2}{Z_0 \cdot dx + R_2} \quad (4)$$

or:

$$Z_0{}^2 \cdot dx - Z_0 R_1 \cdot dx^2 - R_1 R_2 \cdot dx = 0 \quad (5)$$

Leaving out the second order term dx^2, we have:

$$Z_0 = \sqrt{R_1 R_2} = R \quad \text{if } R_1 = R_2 \quad (6)$$

as we can obtain by integration of the distributed line differential equation when the model of fig.3 is taken into account.

This result is quite different to that obtained above for the discrete line and given by (3).

So we point out that the intrinsic difference between the continuous line and the discrete one is based on the second order infinitesimal missing term.

Indeed the feature that a discrete line is infinite does not imply that the line becomes distributed.

III. Case of L-L and C-C LN

Fig. 4 shows a L-L single cell closed with its characteristic impedance Zo

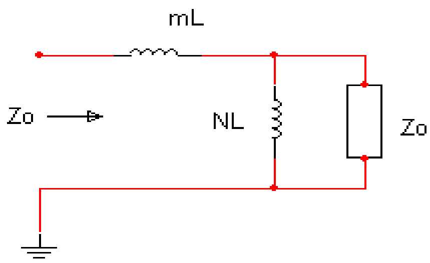


Fig. 4 - A L-L single cell closed by its characteristic impedance Z_0

As before the adopted procedure allow us in the Laplace domain (s=jω) the determination of Z_0:

$$Z_0 = s \cdot mL + \frac{s \cdot mL \cdot Z_0}{s \cdot mL + Z_0}$$

whose positive solution is:

$$Z_0 = s \cdot mL \cdot \frac{1 + \sqrt{1 + 4\frac{n}{m}}}{2}$$

In the particular case, when $n = m$,we have $Zo = sL\phi$ Where ϕ is the golden ratio that will be discussed later.

Let us consider now the following single cell made by only two capacitors (fig.5)

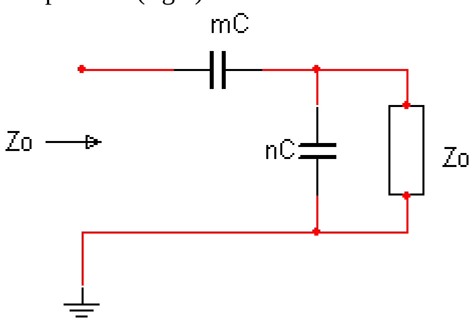


Fig.5:A C-C single cell closed on its characteristic impedance Z0

For this circuit we obtain:

$$Z_0 = \frac{1}{s \cdot mC} + \frac{\frac{1}{s \cdot nC} \cdot Z_0}{\frac{1}{s \cdot nC} + Z_0}$$

Whose positive solution is:

$$Z_0 = \frac{1}{s \cdot mC} \cdot \frac{1 + \sqrt{1 + 4\frac{m}{n}}}{2}$$

In the particular case, when $n = m$ we have Z_0= =ϕ/smC.

IV. Connections among LNs, the "golden section" and the Fibonacci sequence.

Let us consider a LN made by all equal longitudinal resistors kR and transversal resistors lR, whose single cell is shown in Fig.6 with the impedance Z_0 representing the matched load at the end of the LN,

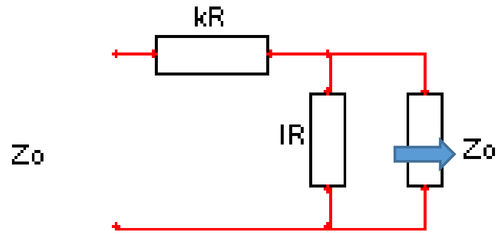


Fig.6 - kR-lR single cell with the characteristic impedance Z_0.

In this case the positive characteristic load impedance is expressed as:

$$Z_0 = \frac{kR}{2} \cdot \left[1 + \sqrt{1 + 4\frac{l}{k}}\right] \tag{7}$$

This result reveals the connection among the LNs, the "golden section" and the "Fibonacci sequence".In fact, from eq. 2, when Z_l= Z_2 = R and $l = k = 1$, we have:

$$Z_0^2 - RZ_0 - R^2 = 0 \tag{8}$$

that can be rewritten as the following proportion:

$$R : Z_0 = Z_0 : R + Z_0 \tag{9}$$

In this form and assuming R=1Ω, eq.9 looks like the second (11) of the following two proportion relationships:

1st1:X =X :1-Xwhose solutions are: 0.618...and -1.618... (10)

2'ld1:X=X:1+X whose solutions are: -0,618...and 1.618... (11)

Along this paper we will call Φ as the "golden ratio", equal to the irrational number: 1.61803398...,$\frac{1}{\Phi}$ the "golden section" equal to the irrational number: 0.61803398...which we indicate in the followingas α and Φ^2equal to: 2.61803398...

These three numbers are really unique. They have the same decimals! This property will be briefly demonstrated in appendix A1. Here we recall some other known particular properties of these numbers:

Φ is the value of Z_0given by the formula (7) when we assume that $R = 1\Omega$ and $l = k = 1$, i.e. the positive solution of the equation

$$\phi^2 - \phi - 1 = 0 \tag{12}$$

So we have directly that:

$$\phi^2 = \phi + 1 \text{or} \phi^2 - 1 = \phi \tag{13}$$

And

$$\phi - 1 = \frac{1+\sqrt{5}}{2} - 1 = \frac{\sqrt{5}-1}{2} = \frac{(\sqrt{5}-1)(\sqrt{5}+1)}{2(\sqrt{5}+1)}$$
$$= \frac{2}{\sqrt{5}+1} = \frac{1}{\phi} = \alpha \tag{14}$$

$$\alpha^{2n} - \alpha^{2n+2} = \frac{1}{\phi^{2n}} - \frac{1}{\phi^{2n+2}} = \frac{\phi^2 - 1}{\phi^{2n+2}} = \frac{\phi}{\phi^{2n+2}} = \frac{1}{\phi^{2n+1}}$$
$$= \alpha^{2n+1} \tag{15}$$

$$2 - \phi = 1 + 1 - \phi = 1 - \frac{1}{\phi} = \frac{\phi - 1}{\phi} = \frac{1}{\phi^2}$$
$$= \alpha^2 \tag{16}$$

We note that Φ^2has a value relatively closed to the basic neperiannumber e = 2.7172... which appears, as well know,in thesolutions of the second order differential equation governing continuous transmission lines.Along this paper we will use the F pointerrelated to the vector of Fibonacci numbers as follows:

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}	F_{16}
0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	987	1597

……….(17)

It is worth pointing out that the two positive solutions of eq.8correspond to the following two limits

$$\Phi = 1{,}618\ldots = \lim_{n\to\infty} \frac{F_{n+1}}{F_n} \; ; \alpha = 0{,}618\ldots = \lim_{n\to\infty} \frac{F_n}{F_{n+1}} \tag{18}$$

The circumstance that in a LN matched load we find, in the particular case of resistive cells, the concept of proportion reveals the intrinsic mechanism of directly coupled cells forming the LN. In fact also all the Fibonacci numbers are directly coupled each other by the following recursive relationship:

$$F_n = F_{n-1} + F_{n-2} \tag{19}$$

And each Fibonacci number is a quasi-middle proportional between the preceding number and the subsequent one, unless an error of plus or minus one, whose relative importance decreases increasing the number itself:

$$F_n{}^2 \cong F_{n-1} \cdot F_{n+1} (\text{with an error equal to } (-1)^n) \tag{20}$$

Further, if we consider the following sequence of non-matched networks having increasing number of not matched resistive cells with$l = k = 1$ and also $V_g = 1$ and R=1 and shown in fig.7,We obtain the following sequence for the biasing current I_g

$$I_g = \left\{\frac{1}{2}, \frac{3}{5}, \frac{8}{13}, \ldots \ldots\right\} \tag{21}$$

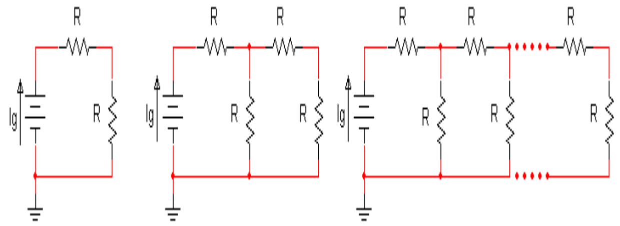


Fig 7: Currents of the supply source in a sequence of LNs with increasing number of cells.

If for instance we increase the number of cells, we get: for four cells LN,$Ig = \frac{21}{34}$A,for the five cells LN$Ig = \frac{55}{89}A$ and so on.

So we can say that in this kind of resistive LN, the ratio among a couple of successive Fibonacci numbers gives the values of the biasing currents I_g. Further, if the biasing voltage is 1 Volt, we have that the input impedance of the cascade of cells is exactly the reciprocal of the ratio which gives the said currentI_g, i.e. $Z_0 = 1/I_g$. Then as example for the four cells LN, $Z_0 = \frac{34}{21}\Omega$and, for the five cells LN, $Z_0 = \frac{89}{55}\ \Omega$.

A similar "Fibonacci-like" behavior is found if we consider the general case in which $l \neq k$assuming the ratio $\frac{l}{k}$as equal to ρ , for a simplification of the matter. If, again, $R = 1$, we obtain as values of the above currents, the following sequence of ratios of Fibonacci-like polynomials:

$$I_g = \left\{\frac{1}{\rho+1}, \frac{2\rho+1}{\rho^2+3\rho+1}, \frac{3\rho^2+4\rho+1}{\rho^3+6\rho^2+5\rho+1}, \dots \dots .. \right\} \qquad (22)$$

And the coefficients of the polynomials at the denominators of sequence (22) can be easily determined remembering the so-called DFF triangle [ ]; see appendix 2

The sequences (21) and (22) are the same if $\rho = 1$. However, if this is not the case, a factor ρmust be introduced in the relation (19) between Fibonacci numbers obtaining a new "Fibonacci-like" sequence expressed as:

$$F^{(\rho)}{}_n = F^{(\rho)}{}_{n-1} + \rho \cdot F^{(\rho)}{}_{n-2} \qquad (23)$$

The following Table I gives the "Fibonacci-like" sequences for the integer values ρ in the range (1 ÷ 10).

As above let us consider a four cells LN, $(n = 8)$, ρ = 4, so $F_n^{(\rho)} = 441$ and$441 = 181 + 4 \cdot 65$; $F_{n+1}^{(\rho)} = 1165$ and $1165 = 441 + 4 \cdot 181$. The current $I_g$is $I_g = \frac{441}{1165}\ A$ and the input impedance is:

$Z_0 = \frac{1165}{441}\Omega$.

Table 1: Orizontalaxes from left to right = n; vertical axes from top to bottom = ρ (from1 to 10)

0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	987	1597	2584	4181
0	1	1	3	5	11	21	43	85	171	341	683	1365	2731	5461	10923	21845	43691	87381	174763
0	1	1	4	7	19	40	97	217	508	1159	2683	6160	14209	32689	75316	173383	399331	919480	2117473
0	1	1	5	9	29	65	181	441	1165	2929	7589	19305	49661	126881	325525	833049	2135149	5467345	14007941
0	1	1	6	11	41	96	301	781	2286	6191	17621	48576	136681	379561	1062966	2960771	8275601	23079456	64457461
0	1	1	7	13	55	133	463	1261	4039	11605	35839	105469	320503	953317	2876335	8596237	25854247	77431669	232557151
0	1	1	8	15	71	176	673	1905	6616	19951	66263	205920	669761	2111201	6799528	21577935	69174631	220220176	704442593
0	1	1	9	17	89	225	937	2737	10233	32129	113993	371025	1282969	4251169	14514921	48524273	164643641	552837825	1869986953
0	1	1	10	19	109	280	1261	3781	15130	49159	185329	627760	2295721	7945561	28607050	100117099	357580549	1258634440	4476859381
0	1	1	11	21	131	341	1651	5061	21571	72181	287891	1009701	3888611	13985621	52871731	192727941	721445251	2648724661	9863177171

However, if the ratio ρ is less than onethe rule expressed by (19) must be modified: in this case the rule is different for the terms of the sequence having even or odd indexes:

$$F_{2n}^{(\rho)} = F_{2n-1}^{(\rho)} \cdot \frac{1}{\rho} + F_{2n-2}^{(\rho)} \quad ; F_{2n+1}^{(\rho)} = F_{2n}^{(\rho)} + F_{2n-1}^{(\rho)} \qquad (24)$$

The following Table II gives the "Fibonacci-like" sequences for the ρ values in the range (1-1/10).

As a particular example let us consider $n = 4, \rho = 1/2$, so$F_{2n}^{(\rho)} = 112$ and $112 = 2x41 + 30$ for the first relation (24);$F_{2n+1} = 153$ and 153=112+41 according to the second relation (24). Again $F_{2n+2} = 418 = 2 \cdot 153 + 112$and $F_{2n+3} = 418 + 153 = 571$. So the source current of the four cell LN is $I_g = \frac{112}{153}\ A$ and the inputi impedance is $Z_0 = \frac{153}{112}\ \Omega$. And for a LN of five cells we have $I_g = \frac{418}{571}\ A$ and $Z_0 = \frac{571}{418}\ \Omega$

Table 2:Orizontal axes from left to right = n; vertical axes from top to bottom = ρ(from 0.1 to 1)

$\Pi \rightarrow$

0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	987	1597	2584	4181
0	1	2	3	8	11	30	41	112	153	418	571	1560	2131	5822	7953	21728	29681	81090	110771
0	1	3	4	15	19	72	91	345	436	1653	2089	7920	10009	37947	47956	181815	229771	871128	1100899
0	1	4	5	24	29	140	169	816	985	4756	5741	27720	33461	161564	195025	941664	1136689	5488420	6625109
0	1	5	6	35	41	240	281	1645	1926	11275	13201	77280	90481	529685	620166	3630515	4250681	24883920	29134601
0	1	6	7	48	55	378	433	2976	3409	23430	26839	184464	211303	1452282	1663585	11433792	13097377	90018054	103115431
0	1	7	8	63	71	560	631	4977	5608	44233	49841	393120	442961	3493847	3936808	31051503	34988311	275969680	310957991
0	1	8	9	80	89	792	881	7840	8721	77608	86329	768240	854569	7604792	8459361	75279680	83739041	745192008	828931049
0	1	9	10	99	109	1080	1189	11781	12970	128511	141481	1401840	1543321	15291729	16835050	166807179	183642229	1819587240	2003229469
0	1	10	11	120	131	1430	1561	17040	18601	203050	221651	2419560	2641211	28831670	31472881	343560480	375033361	4093894090	4468927451

V. COMPARISON BETWEEN MATCHED AND NON-MATCHED LNS.

A. *Non matched LNs*

Let we start by considering the features of the non-matched resistive LNs as shown in fig. 8.

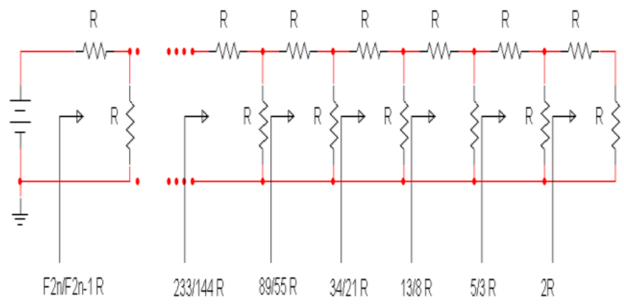


Fig.8–Resistive LN not closed with its characteristic impedance Z0: impedance values in the indicated points.All the impedances are considered towards the right as indicated by arrows.

In these LNs we point out the role played by Fibonacci numbers,which allow the straightforward determination of all the impedances at any point and all the voltages at any node as well. Note that the impedances of the LN of Fig. 8 are, obviously, if we pose R=1, the reciprocal of the currents. As it is easy to prove [IEEE], all the nodes in fig. 8have voltages expressed by the ratio of two Fibonacci numbers. Indeed, starting from the voltage source, i.e. from the node n, we have:

$$I_n = \frac{V_n}{Z_n} = V_g \cdot \frac{F_{2n-1}}{F_{2n} \cdot R} \quad (25)$$

and:

$$V_{n-1} = V_n - I_n \cdot R = V_g \cdot \left(1 - \frac{F_{2n-1}}{F_{2n}}\right) = V_g \frac{F_{2n} - F_{2n-1}}{F_{2n}} = V_g \frac{F_{2n-2}}{F_{2n}} \quad (26)$$

Then, using an iterative procedure:

$$I_k = V_g \frac{F_{2n-2k+1}}{F_{2n} \cdot R}; \ V_{n-k} = V_g \frac{F_{2n-2k}}{F_{2n}} \quad (27)$$

Formulae (27) allow the direct determination of any node voltage and any current of longitudinal branch, and by a straightforward procedure also currents in any transversal branch.The procedure is simple: only the Fibonacci numbers and their pointers are sufficient to solve the LN represented in fig.8 whatever the number of cells can be.

Increasing the number of cells the voltages along the R-R LN approximate better and better the power of the golden section $\alpha = l/\phi$.. In fact we have that:

$V_k \, (l/\phi^2)^k$ where k = 1,2,3,..,N indicate the number of cells ....(28)

$V_k \phi^{-2k} = (\phi^2)^{-k} = \alpha^{2k}$, with α = 0,61803398.............(29)

The differences between V_K and α^{2k} are relatively small, but never zero and this is conceptually important when matched LNs are taken into consideration, as we will see in the following.

A simple expression for the node voltages (see fig.8) of the non-matched resistive LN is the following:

$V_{n,8} = F_{2(N-n)+1}/F_{2N+1}$……… ………………………… (30)

where N does represent the total number of cells and n goes from 1 to N.

In practice, once the number N of cells has been chosen, let us suppose 8, as an example, we use as denominator (F_{2N+1}) which corresponds to F_{17} =1597and, as far as the numerator is concerned, it starts from $F_1$and only considers the odd numbers till the last one correspondent to F_{15} = 610

We give here the node voltages determined by the above-illustrated method. These results have been confirmed by experiments, of course in a first approximation, essentially due to the experimental uncertainties related to the resistance values.

The exact voltage values are given, with reference to fig. 8 without the characteristic impedanceZ_0, by:

$$V_{1,8} = F_{15}/F_{17} = \frac{610}{1597}$$

$$V_{2,8} = F_{13}/F_{17} = \frac{233}{1597}$$

$$V_{3,8} = F_{11}/F_{17} = \frac{89}{1597}$$

$$V_{4,8} = F_9/F_{17} = \frac{34}{1597}$$

$$V_{5,8} = F_7/F_{17} = \frac{13}{1597}$$

$$V_{6.8} = F_5/F_{17} = \frac{5}{1597}$$

$$V_{7,8} = F_3/F_{17} = \frac{2}{1597}$$

$$V_{8,8} = F_1/F_{17} = \frac{1}{1597}$$

As we can see, only Fibonacci numbers are used for the above node voltage expressions as numerator and denominator

B. *Matched LNs:Case of resistances all equal to each other* $(\varrho = 1)$

In this case, by assuming that $k = l = 1$ in eq.7 we obtain:

$$Z_0 = R\frac{1+\sqrt{5}}{2} = R\phi$$

which corresponds to the input impedance of the entire infinite LN and also the same impedance that each cell views as input impedance of the remaining of the LN, as reported in figure 9, where is also indicated the impedance viewed in the middle of the cell, say

$Z_0' = Z_0 - R = R(\phi - 1) = R/\phi$.

Also in the resistive LN closed on its characteristic impedance Z_0(fig.9) the node voltages are once again easily determined, however not through the ratio of two Fibonacci numbers: instead it can be shown that these voltage values, if $R = 1\,\Omega$ and the driving voltage is $1\,V$, are exactly expressed by theeven powers of the golden section α, where $\alpha = 1/\phi$ i.e.: (α^2,α^4 α^6, …. $\alpha16$), as in fig.9.Further, since the driving voltage is the sum of the entire voltage differences between two consecutive nodes and the residual voltage across Z_0, we can write:

$$\sum_{n=0}^{8}\left(\frac{1}{\phi^{2n}}-\frac{1}{\phi^{2n+2}}\right)+\frac{1}{\phi^{8}}=1 \qquad (31)$$

Or, taking account that the presence of the termination Z_0 which renders infinite the LN:

$$\sum_{n=1}^{\infty}\left(\frac{1}{\phi^{2n}}-\frac{1}{\phi^{2n+2}}\right)=\sum_{n=1}^{\infty}\left(\frac{\phi^{2}-1}{\phi^{2n+2}}\right)=\sum_{n=1}^{\infty}\left(\frac{\phi}{\phi^{2n+2}}\right)=\sum_{n=1}^{\infty}\left(\frac{1}{\phi^{2n+1}}\right)$$
$$=\sum_{n=1}^{\infty}\alpha^{2n+1}=1 \qquad (32)$$

We note that in the infinite LN, as in fig.9, the voltages and the currents of each cell are middle proportional (MP) with respect to the homologue quantities of the preceding and successive cell respectively:

the voltage across the longitudinal branch:

$$(\alpha^{2n+2}-\alpha^{2n+4})^2=(\alpha^{2n}-\alpha^{2n+2})(\alpha^{2n+4}-\alpha^{2n+6})$$

the current of the longitudinal branch:

$$\left(\frac{\alpha^{2n+2}-\alpha^{2n+4}}{R}\right)^2=\left(\frac{\alpha^{2n}-\alpha^{2n+2}}{R}\right)\left(\frac{\alpha^{2n+4}-\alpha^{2n+6}}{R}\right)$$

the voltage across the transversal branch:

$$(\alpha^{2n+2})^2=\alpha^{2n}\cdot\alpha^{2n+4}$$

the current of the transversal branch:

$$\left(\frac{\alpha^{2n+2}}{R}\right)^2=\frac{\alpha^{2n}}{R}\cdot\frac{\alpha^{2n+4}}{R}$$

And also the current of the transversal branch is MP among the currents of the longitudinal branches of the preceding and successive cell, recalling property (15) and referring to figure 10:

$$\frac{(\alpha^{2n+2})^2}{R^2}=\frac{\alpha^{2n}-\alpha^{2n+2}}{R}\cdot\frac{\alpha^{2n+2}-\alpha^{2n+4}}{R}=\frac{\alpha^{2n+1}\alpha^{2n+3}}{R^2}$$
$$=\frac{\alpha^{4n+4}}{R^2}$$

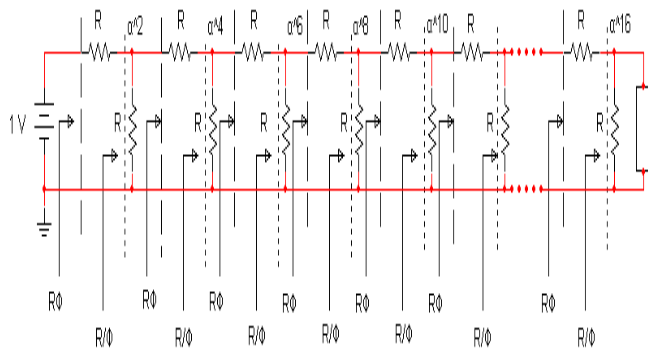


Fig. 9–Infinite discrete resistive LN: the voltages and the impedances along the line are indicated.

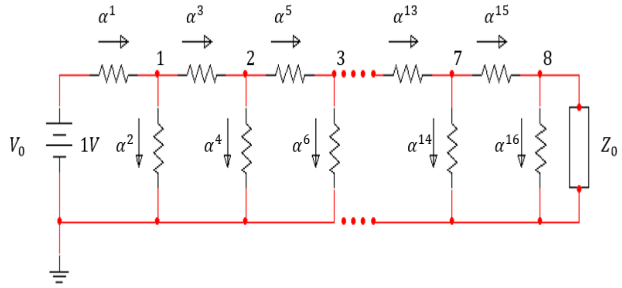


Fig. 10 – Infinite discrete resistive LN: the currents across the branches of the LN are indicate

Remembering that $\alpha=\frac{1}{\phi}$, we have for all the currents of the branches of the resistive LN of figure 10:

$$I_g=I_{01}=\frac{1}{\phi};\ I_{10}=\frac{1}{\phi^2};\ I_{12}=\frac{1}{\phi^3};I_{20}$$
$$=\frac{1}{\phi^4};\ldots\ldots;I_{78}=\frac{1}{\phi^{15}};\ I_{80}$$
$$=\frac{1}{\phi^{16}} \qquad (33)$$

Since the current of the driving source is the sum of all the currents of the transversal branches, we also have that: $\sum_{n=1}^{8}\left(\frac{1}{\phi^{2n}}\right)+I_{Zo}=\frac{1}{\phi}$ (34)

Or, for an infinite LN:

$$\sum_{n=1}^{\infty}\left(\frac{1}{\phi^{2n}}\right)=\frac{1}{\phi} \qquad (35)$$

So that, collecting the two results (28) and (31), we have that:

$$\sum_{n=1}^{\infty}\left(\frac{1}{\phi^{n}}\right)=1+\frac{1}{\phi}=\phi \qquad (36)$$

C. Matched LNs: Case of resistances responding to the following condition :$(\varrho=1)$.

In this case, assuming $k=1$ *and* $k/l=\varrho$ in (7), the characteristic impedance becomes:

$$Z_0=R\frac{1+\sqrt{1+4\varrho}}{2} \qquad (37)$$

and:

$$Z_0{}'=Z_0-R=R\left(\frac{1+\sqrt{1+4\varrho}}{2}-1\right)$$
$$=R\frac{\sqrt{1+4\varrho}-1}{2} \ (38)$$

Choosing for instance $\varrho=\frac{\phi}{4}$, we obtain:

$$Z_0=R\frac{1+\sqrt{1+\phi}}{2}=R\frac{1+\phi}{2}=R\frac{\phi^2}{2}\rightarrow Z_0{}'=\frac{\phi-1}{2}$$
$$=\frac{\alpha}{2} \qquad (39)$$

Some of the values of ϱ that allow the elimination of the radix in (33) and (34) are listed below:

$$\varrho=\frac{3}{4}\rightarrow Z_0=\frac{3}{2}R\ \rightarrow Z_0{}'=\frac{1}{2}R$$
$$\varrho=2\ \rightarrow Z_0=2R\ \rightarrow Z_0{}'=R$$
$$\varrho=6\ \rightarrow Z_0=3R\rightarrow Z_0{}'=2R$$
$$\varrho=12\rightarrow Z_0=4R\rightarrow\ Z_0{}'=3R$$
$$\varrho=20\rightarrow Z_0=5R\ \rightarrow Z_0{}'=4R$$

VI. LNs HAVING TWO LONGITUDINAL BRANCHES AND NOT CLOSED ON ITS CHARACTERISTIC IMPEDANCE.

Another interesting circuit is the double resistive LN that will be briefly taken into consideration with and without the characteristic impedanceZ0.

For this network (fig.11), where, as a particular case, the longitudinal and transversal resistor values are equal to R/2 and R, respectivelywe have determined the differential voltages across allthe transversal resistors. We have found that their values are the same we have found for the LN represented in fig.8.

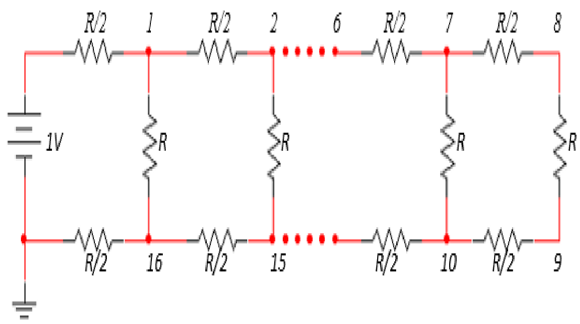


Fig.11 Double resistive LN, non matched at its end with Z0

For fig.11 we have the following transversal voltage values.

$V_{1,16} = 610/1597$
$V_{2,15} = 233/1597$
$V_{3,14} = 89/1597$
$V_{4,13} = 34/1597$
$V_{5,12} = 13/1597$
$V_{6,11} = 5/1597$
$V_{7,10} = 2/1597$
$V_{8,9} = 1/1597$

Also in this case we see a strong link with the Fibonacci numbers which determine the exact values of the differential voltages, but we like to point out that all the above voltages are close but non coincident respectively to; $\alpha^2 \alpha^4$ α^6 $\alpha^8 \alpha^{10} \alpha^{12}$ α^{14} α^{16}

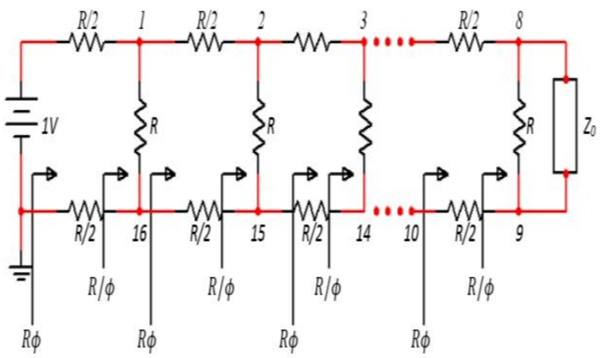


Fig.12: Double resistive LN matched with Zo

It is perhaps useful to plot these values and see the behavior of the voltages along this type of LN. when the comparison is with respect to the natural exponential behavior. See fig 13

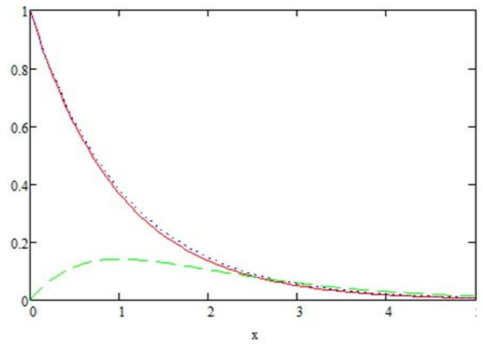


Fig. 13 Behavior of the two described exponentials
___ The classical exponential ex
……..The exponential Φ^x
---- The difference $(\Phi^x - e^x)$

As we can see the difference is relatively small and reaches its maximum value when X is equal to 1.

Another resistive LN.which gives evidence of Fibonacci numbers is that shown in fig.15, whereall the longitudinal resistors have a value equal to R/2, all the transversal resistors have a value equal to R. In this case, the L.N. is terminated by its characteristic impedance Z_0

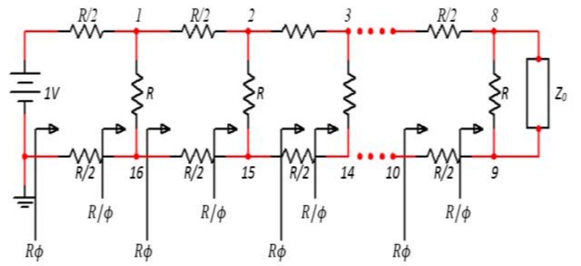


Fig.14: Double resistive LN matched with Zo

The following table N°3 gives the impedance values in the different points signd by capital letters,represented in fig.14 with three different cases (k=1/2,l=1; k=1,l=1/2; k=1,l=1) of longitudinal and transversal impedances.

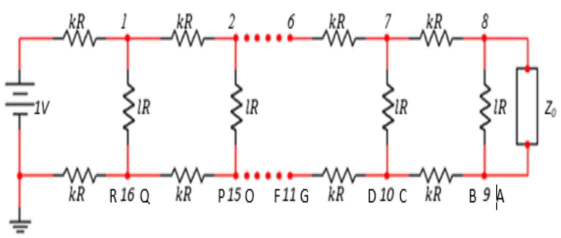


Fig.15: Resistive LN closed with the Characteristic impedance Zo with longitudinal resistor value equal to kR and transversal resistor values equal to lR

Table 3

Position	impedances	impedances	impedances
	k=1/2;l=1	k=1;l=1/2	K=1;l=1
A	Rφ	R(√2 +1)	R(√3+1)
B	R/φ	R(√2-1)	R(√3-1)
C	Rφ	R(√2 +1)	R(√3+1)
D	R/φ	R(√2-1)	R(√3-1)
. . O	R φ	R(√2 +1)	R(√3+1)
P	R/φ	R(√2-1)	R(√3-1)
Q	Rφ	R(√2 +1)	R(√3+1)
R	R/φ	R(√2-1)	R(√3-1)

The following table N°4 gives the node voltage values for the three types of double resistive L.N. closed on their characteristic impedance Z_0

Table 4

Voltages	K=1;l=1	K=1;l=1/2	K=1/2;l=1
V 1,16	(2-√3)^2	(√2 - 1)^2	α ^2
V2,15	(2 - √3)^4	(√2 - 1)^4	α^4
V3,14	(2 - √3)^6	(√2 - 1)^6	α^6
V 4, 13	(2 - √3)^8	(√2 - 1)^8	α^8
V 5,12	(2 - √3)^10	(√2 - 1)^10	α ^10
V 6,11	(2 - √3)^12	(√2 - 1)^12	α ^12
V7, 10	(2 - √3)^14	(√2 - 1)^14	α ^14
V 8, 9	(2 - √3)^16	(√2 - 1)^16	α ^16

The results expressed in the first column of Tab.N°3 show that Z_0 is equal to Rϕ.This value can also be obtained by different values of the longitudinal and transversal resistors values provided that their sum be always equal to R. So, for instance, we can use nR/M and (M-n)R/M, as the values of the two longitudinal branches respectively, taking into account that n<M. In this condition Zo is always equal to $R\phi$. See fig16

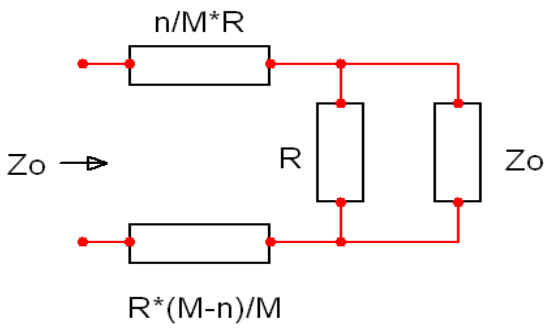
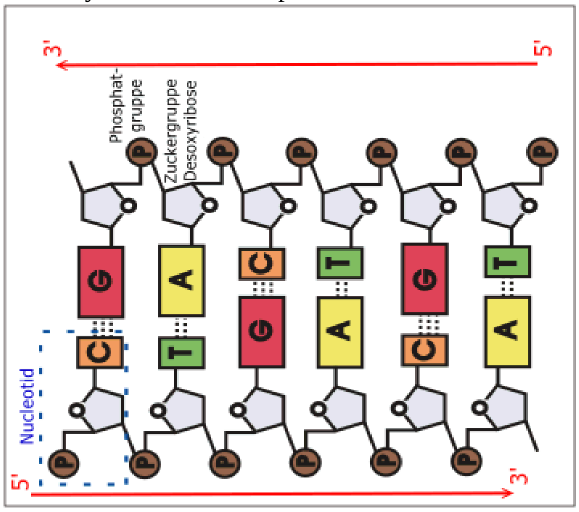


Fig.16: Dual ladder network made by longitudinal impedances whose sum is equal to R

At the end of this work we give a view of a piece of DNA(taken from “Wikipedia” just to show how impressive is its similarity in comparison with the double LN. The double longitudinal structures made by same sugar and phosphate groups look like the double longitudinal impedances in double LN, while the transversal impedances can be divided in only two groups namely CG or GC(three dotted line)and TA or AT(two dotted lines). These two groups could be modeled by two different impedances.



VII. Conclusions

We have shown a large presence of Fibonacci Numbers (FN) in varieties of Ladder Networks (LNs)in the particular case where the longitudinal and transversal impedances are the same. This is also thru not only in the R-R case but also in those case where the single cell is formed by capacitors and inductors.

In non matched R-R LNs we have seen that the voltages in any node is expressed by the ratio of Fibonacci numbers and we know that each FN is “quasi” middle proportional between the preceding and the next.

This indicates that a perfect proportion does exist between the characteristic impedance and the basic element of the LN. The voltages inR-R LNs are perfectly expressed by the ratio of two FN.

In matched R-R or C-C or even L-L LNs the much stronger reason …is related to the fact that the governing equation for the impedance determination, no matter how many single cells are considered, is always of the second order which is the basis of the middle proportion concept.

On the other hand in those LN non closed on their characteristic impedance we observe the presence of the Fibonacci numbers whose construction does not allow a perfect proportion; this only happens with a growing approximation when the number of cells approaches infinity.

We point out that the investigation presented in this work may have some positive impact in those networks representing approximate models of both DNA and RNA structures where so far some problems related to the equivalence between chemical basis and passive R L C components still exist.

Acknowledgment

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Appendix 1

Demonstration of the fact that $1/\varphi$, φ, φ^2 have the same decimals,

Starting from the following relationship:

1,618.... = $\phi = (1 + 1/\phi)$; and squaring both members we get:

$\phi^2 = (1 + 1/\phi)^2$ or:

$\phi^2 = 1 + 2/\phi + 1/\phi^2$ leaving out the denominator we get:

$\phi^4 = (1 + \phi)^2$; taking the root square finally we have:

$\phi^2 = 1 + \phi = 1 + 1{,}618... = 2{,}618...$cvd

Appendix 2

The DFF triangle introduced in ref [2] and reproduced in fig.15 where n(number of cells) and i are restricted to 5, allows to directly write the expression of the generic ladder network transfer function.

n \ i	0	1	2	3	4	5
0	1					
1	1	1				
2	1	3	1			
3	1	6	5	1		
4	1	10	15	7	1	
5	1	15	35	28	9	1

Fig.17 A portion of the DFF triangle

The generic term of this triangle can be expressed as follows:

$$a(n,i) = \binom{n+i}{n-i}$$

Using this expression and the ratio between the longitudinal and transversal impedances k = Z1/Z2 it is possible to express the transfer function as follows:

$Gn(k) = 1/\sum a(n,i)k^i = 1/S_{n,k}$

By this expression the node voltages are simply given by:

$V\beta = S_{n-\beta}(k) / S_n(k)\, V_i$

It is worth pointing out that the sum of the numbers in each row gives again the Fibonacci numbers.

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