

# Path Relatively Prime Cordial Graph

Dr. A. Nellai Murugan ,V.Nishanthini

**Abstract**— Let  $G = (V, E)$  be a graph with  $p$  vertices and  $q$  edges. A Analytic Mean Cordial Labeling of a Graph  $G$  with vertex set is a bijection from  $v = \{1, 2, \dots, p\}$ . The induced edge labelling are defined by  $f(u, v) = 0$  if either  $f(u)$  divides  $f(v)$  (or)  $f(v)$  divides  $f(u)$  one otherwise and if any one of the vertex label is  $1$ , the induced edge label is  $0$ .

The graph that admits a Relatively Prime Cordial Graph is called Relatively Prime Cordial Graph. In this paper, we proved that path related graphs  $P_n, P_n \odot K_1, P_n^2, P_2: S_n, P_n + K_1$  are Relatively Prime Cordial Graph.

**Index Terms**— Relatively Prime Cordial Graph, Relatively Prime Cordial Labeling.

## I. INTRODUCTION

A graph  $G$  is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of  $G$  which is called edges. Each pair  $e = \{u, v\}$  of vertices in  $E$  is called an edge or a line of  $G$  in which  $e$  is said to join  $u$  and  $v$ . We write  $e = uv$  and say that  $u$  and  $v$  are adjacent vertices (sometimes denoted as  $u \text{ adj } v$ ); vertex  $u$  and the edge  $e$  are incident with each other, as are  $v$  and  $e$ . If two distinct edges  $e_1$  and  $e_2$  are incident with a common vertex, then they are called *adjacent edges*. A graph with  $p$  vertices and  $q$  edges is called  $(p, q)$  - graph. In this paper, we proved that path related graphs  $P_n, P_n \odot K_1, P_n^2, P_2: S_n, P_n + K_1$  are Relatively Prime Cordial Graph. For graph theory terminology, we follow [2].

## II. PRELIMINARIES

Let  $G = (V, E)$  be a graph with  $p$  vertices and  $q$  edges. A Analytic Mean Cordial Labeling of a Graph  $G$  with vertex set is a bijection from  $v = \{1, 2, \dots, p\}$ . The induced edge labelling are defined by  $f(u, v) = 0$  if either  $f(u)$  divides  $f(v)$  (or)  $f(v)$  divides  $f(u)$  one otherwise and if any one of the vertex label is  $1$ , the induced edge label is  $0$ .

The graph that admits a Relatively Prime Cordial Graph is called Relatively Prime Cordial Graph. In this paper, we proved that path related

graphs  $P_n, P_n \odot K_1, P_n^2, P_2: S_n, P_n + K_1$  are Relatively Prime Cordial Graph.

**Definition: 2.1**  $P_n$  is a path of length  $n - 1$ .

**Definition: 2.2** The Corona  $= G_1 \odot G_2$  of two graph  $G_1$  and  $G_2$  is defined as the graph  $G$  obtained by taking one copy of  $G_1$  (which has  $P_1$  points) and  $P_1$  copies of  $G_2$  and joining the  $i^{\text{th}}$  point of  $G_1$  to every point in the  $i^{\text{th}}$  copy of  $G_2$ . The graph  $P_n \odot K_1$  is called a *comb*.

**Definition: 2.3**

$(P_n^2)$  is a path of length  $n-1$  of twice.

**Definition: 2.4**

Star of length one is joined with every vertex of a path  $P_n$  through an edge. It is denoted by  $P_2: S_n$

**Definition: 2.5**

The join of  $G_1$  and  $G_2$  is the graph  $G = G_1 + G_2$  with vertex set  $V = V_1 \cup V_2$  and edge set  $E = E_1 \cup E_2 \cup \{uv : u \in V_1, v \in V_2\}$ . The graph  $P_n + K_1$  is called a Fan.

## II. MAIN RESULTS

**THEOREM: 3.1**

Path  $P_n$  is Relatively Prime Cordial Graph.

**Proof:**

Let  $G$  be  $P_n$

Let  $V(G) = \{u_i : 1 \leq i \leq n\}$

Let  $E(G) = \{(u_i, u_{i+1}) : 1 \leq i \leq n - 1\}$

Define  $f : V(G) \rightarrow \{1, 2, \dots, p\}$

The vertex labeling are

When  $n = \text{even}$

$$f(u_i) = 2i \quad 1 \leq i \leq \frac{n}{2}$$

$$f(u_{n+1-i}) = 2i - 1 \quad 1 \leq i \leq \frac{n}{2}$$

When  $n = \text{odd}$

$$f(u_i) = 2i \quad 1 \leq i \leq \frac{n-1}{2}$$

$$f(u_{n+1-i}) = 2i - 1 \quad 1 \leq i \leq \frac{n+1}{2}$$

The induced edge labeling are,

When  $n = \text{even}$

$$f^*(u_{n-1}u_n) = 0$$

$$f^*(u_i u_{i+1}) = 0 \quad 1 \leq i \leq \frac{n-2}{2}$$

$$f^*(u_i u_{i+1}) = 1 \quad \frac{n}{2} \leq i \leq n - 2$$

When  $n = \text{odd}$

$$f^*(u_{n-1}u_n) = 0$$

Dr. A.Nellai Murugan, Dept. of Mathematics, V.O. Chidambaram College, Tuticorin.

V.Nishanthini, Dept. of Mathematics, V.O. Chidambaram College, Tuticorin.

$$f^*(u_i u_{i+1}) = 0 \quad 1 \leq i \leq \frac{n-2}{2}$$

$$f^*(u_i u_{i+1}) = 1 \quad \frac{n-1}{2} \leq i \leq n-2$$

Here, When  $n = 2m, m > 1$

$$e_f(0) = m$$

$$e_f(1) = m - 1$$

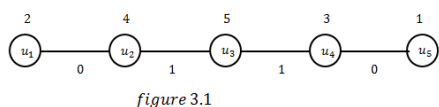
$$n = 2m + 1, m > 1$$

$$e_f(0) = m = e_f(1)$$

$$|e_f(1) - e_f(0)| \leq 1$$

Hence, Path  $P_n$  is Relatively Prime Cordial Graph.

For example, The Relatively Prime Cordial Graph of  $P_5$  are shown in the figure



**THEOREM: 3.2**

$P_n \odot K_1$  is a Relatively Prime Cordial Graph.

**Proof:**

Let  $G$  be  $P_n \odot K_1$

Let  $V(G) = \{ (u_i, v_i) : 1 \leq i \leq n \}$

Let

$$E(G) = \{ [(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i v_i) : 1 \leq i \leq n] \}$$

Define  $f : V(G) \rightarrow \{1, 2, \dots, p\}$

The vertex labeling are,

$$f(u_i) = 2i \quad 1 \leq i \leq n$$

$$f(v_i) = 2i - 1 \quad 1 \leq i \leq n$$

The induced edge labeling are,

$$f^*(u_i u_{i+1}) = 0 \quad 1 \leq i \leq n-1$$

$$f^*(u_1 v_1) = 0$$

$$f^*(u_i v_i) = 1 \quad 2 \leq i \leq n$$

Here, When  $n = m, m > 1$

$$e_f(0) = m$$

$$e_f(1) = m - 1$$

$$|e_f(1) - e_f(0)| \leq 1$$

Hence,  $P_n \odot K_1$  is Relatively Prime Cordial Graph.

For example, Relatively Prime Cordial Graph,

$P_5 \odot K_1$  are shown in the fig

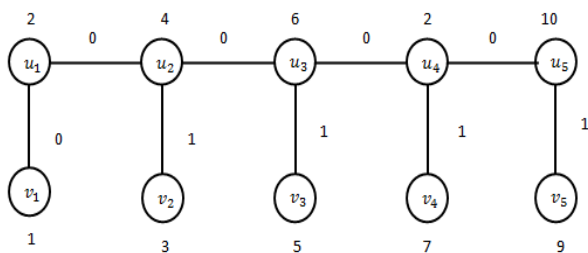


figure 3.2

**THEOREM: 3.3**

Graph  $Pn^2$  is a Relatively Prime Cordial Graph.

**Proof:**

Let  $G$  be  $Pn^2$  Graph

Let  $V(G) = \{ u_i, : 1 \leq i \leq n \}$

Let

$$E(G) = \{ [(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i u_{i+2}) : 1 \leq i \leq n-2] \}$$

Vertex Labeling:

When  $n = \text{even}$

$$f(u_i) = 2i \quad 1 \leq i \leq \frac{n}{2}$$

$$f(u_i) = 2i - 1 \quad \frac{n+2}{2} \leq i \leq n$$

When  $n = \text{odd}$

$$f(u_i) = 2i \quad 1 \leq i \leq \frac{n-1}{2}$$

$$f(u_{n+1-i}) = 2i - 1 \quad 1 \leq i \leq \frac{n+1}{2}$$

Edge Labeling:

When  $n = \text{even}$

$$f^*(u_n u_{n-1}) = 0$$

$$f^*(u_i u_{i+1}) = 0 \quad 1 \leq i \leq \frac{n-2}{2}$$

$$f^*(u_i u_{i+1}) = 1 \quad \frac{n}{2} \leq i \leq n-2$$

$$f^*(u_{n-2} u_n) = 0$$

$$f^*(u_i u_{i+2}) = 0 \quad 1 \leq i \leq \frac{n-4}{2}$$

$$f^*(u_i u_{i+2}) = 1 \quad \frac{n-2}{2} \leq i \leq n-2$$

When  $n = \text{odd}$

$$f^*(u_i u_{i+1}) = 0 \quad 1 \leq i \leq \frac{n-3}{2}$$

$$f^*(u_i u_{i+1}) = 1 \quad \frac{n-1}{2} \leq i \leq n-2$$

$$f^*(u_{n-1} u_n) = 0$$

$$f^*(u_{n-2} u_n) = 0$$

$$f^*(u_i u_{i+2}) = 0 \quad 1 \leq i \leq \frac{n-3}{2}$$

$$f^*(u_i u_{i+2}) = 1 \quad \frac{n-1}{2} \leq i \leq n-3$$

When  $n = 2m + 1, m > 1$

$$e_f(0) = 2m - 1$$

$$e_f(1) = 2m$$

Hence, Path  $P5^2$  is a Relatively Prime Cordial Graph.

For example, The Relatively Prime Cordial Graph of  $P5^2$  are shown in the figure

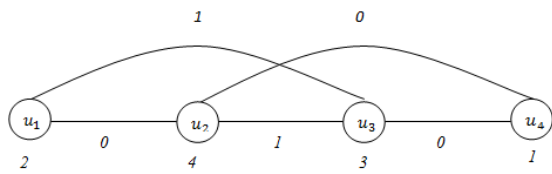


figure 3.3

**THEOREM: 3.4**

Graph  $(P_2: S_n)$  is a Relatively Prime Cordial Graph.

**Proof:**

Let

$$V(p_2: s_n) = \{ u_i, : 1 \leq i \leq 4; u_{1i}, u_{2i}, 1 \leq i \leq n \}$$

Let

$$E(p_2: s_n) = \{ [ u_3 u_4 ] U [ u_1 u_4 ] U [ u_2 u_3 ] U [ u_1 u_{1i} : 1 \leq i \leq n ] U [ u_2 u_{2i} : 1 \leq i \leq n ] \}$$

Define  $f : V(G) \rightarrow \{1, 2, \dots, p\}$

Vertex Labeling:

$$\begin{aligned} f(u_i) &= i & 1 \leq i \leq 4 \\ f(u_{1i}) &= 2i + 4 & 1 \leq i \leq n \\ f(u_{2i}) &= 2i + 3 & 1 \leq i \leq n \end{aligned}$$

Edge labeling:

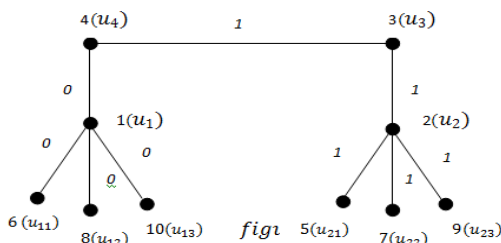
$$\begin{aligned} f^*(u_i u_{i+1}) &= 1 & 2 \leq i \leq 3 \\ f^*(u_1 u_4) &= 0 \\ f^*(u_1 u_{1i}) &= 0 & 1 \leq i \leq n \\ f^*(u_2 u_{2i}) &= 1 & 1 \leq i \leq n \end{aligned}$$

Here, When  $n = m$

$$\begin{aligned} e_f(0) &= n + 1 \\ e_f(1) &= n + 2 \\ e_f(0) + 1 &= e_f(1) \\ |e_f(1) - e_f(0)| &\leq 1 \end{aligned}$$

Hence,  $P_2: S_n$  is Relatively Prime Cordial Graph.

For example, The Relatively Prime Cordial Graph of  $P_2: S_n$  are shown in the figure



figi

**THEOREM: 3.5**

Graph  $P_n + K_1$  is Relatively Prime Cordial graph.

**Proof:**

Let  $p_n + k_1, n > 1$

$$Let V(p_n + k_1) = \{u, u_i : 1 \leq i \leq n\}$$

Let

$$E(p_n + k_1) = \{ [uu_i]; 1 \leq i \leq n \} \cup \{ [u_i u_{i+1}]; 1 \leq i \leq n - 1 \}$$

Define  $f : V(G) \rightarrow \{1, 2, \dots, p\}$

Vertex Labeling:

$$\begin{aligned} f(u) &= 1 \\ f(u_i) &= i + 1 & 1 \leq i \leq n \end{aligned}$$

Edge Labeling:1

$$\begin{aligned} f^*(uu_i) &= 1 \\ f^*(u_i u_{i+1}) &= 1 & 1 \leq i \leq n - 1 \end{aligned}$$

Here, When  $n = m, m > 1$

$$\begin{aligned} e_f(0) &= m \\ e_f(1) &= m - 1 \end{aligned}$$

$$|e_f(1) - e_f(0)| \leq 1$$

Hence,  $P_n + K_1$  is Relatively Prime Cordial Graph.

For example, The Relatively Prime Cordial Graph of  $P_n + K_1$  are shown in the figure

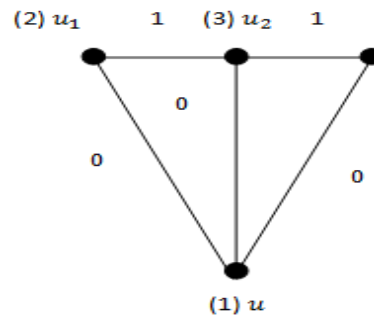


figure 3.5

REFERENCES

- [1] Gallian J.A, A Dynamic Survey of Graph Labelling, The Electronic Journal of Combinatorics, 6 (2001) #DS6
- [2] Harry. F, Graph Theory, Adadison-Wesley Publishing Company Inc, USA, 1969
- [3] A.Nellai Murugan and V.Baby Suganya, A study on cordial labeling of Splitting Graphs of star Attached C3 and (2k+1)C3 ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 142 -147. I.F 6.531
- [4] A.Nellai Murugan and V.Brinda Devi , A study on Star Related Divisor cordial Graphs ,ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 169-172. I.F 6.531
- [5] A.Nellai Murugan and M. Taj Nisha, A study on Divisor Cordial Labeling Star Attached Path Related Graphs, ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 173-178. I.F 6.531
- [6] A. Nellai Murugan and V .Sripratha, Mean Square Cordial Labelling , International Journal of Innovative Research & Studies, ISSN 2319-9725 ,Volume 3, Issue 10Number 2 ,October 2014, PP 262-277.
- [7] A.Nellai Murugan and A.L Poornima ,Meanness of Special Class Of Graphs, International Journal of Mathematical Archive. ISSN 2229-5046, Vol 5 , issue 8, 2014, PP 151-158.
- [8] A.Nellai Murugan and A.Mathubala, Path Related Homo- cordial graphs, International Journal of Innovative Science, Engineering & Technology , ISSN 2348-7968, Vol.2, Issue 8 ,August. 2015, PP 888-892. IF 1.50, IBI-Factor 2.33
- [9] A.Nellai Murugan ,V.Selva Vidhya and M Mariasingam, Results On Hetro- cordial graphs, International Journal of Innovative Science, Engineering & Technology , ISSN 2348-7968, Vol.2, Issue 8 ,August. 2015, PP 954-959. IF 1.50, IBI-Factor 2.33

- [10] A.Nellai Murugan , and R.Megala, Path Related Relaxed Cordial Graphs , International Journal of Scientific Engineering and Applied Science (IJSEAS) - ISSN: 2395-3470,Volume-1, Issue-6, September ,2015 ,PP 241-246 IF ISRA 0.217.
- [11] A.Nellai Murugan and J.Shiny Priyanka, Tree Related Extended Mean Cordial Graphs, International Journal of Research -Granthaalayah, ISSN 2350-0530,Vol.3, Issue 9 ,September. 2015, PP 143-148. I.F . 2.035(I2OR).
- [12] A.Nellai Murugan and S.Heerajohn, Cycle Related Mean Square Cordial Graphs, International Journal Of Research & Development Organization – Journal of Mathematics,Vol.2, Issue 9 ,September. 2015, PP 1-11.
- [13] A.Nellai Murugan and A.Mathubala, Special Class Of Homo- Cordial Graphs, International Journal Emerging Technologies in Engineering Research, ISSN 2524-6410,Vol.2, Issue 3 ,October 2015, PP 1-5.
- [14] A.Nellai Murugan , and R.Megala ,Tree Related Relaxed Cordial Graphs, International Journal of Multi disciplinary Research & Development , ISSN: 2349-4182,Volume-2, Issue-10, October ,2015 ,PP 80-84 . IF 5.742.
- [15] A.Nellai Murugan , and A.Mathubala ,Cycle Related Homo-Cordial Graphs , International Journal of Multi disciplinary Research & Development, ISSN: 2349-4182,Volume-2, Issue-10, October ,2015 ,PP 84-88 . IF 5.742.
- [16] L. Pandiselvi , A. Nellai Murugan , and S. Navaneethakrishnan , Some Results on Near Mean Cordial , Global Journal of Mathematics , ISSN 2395-4760,Volume 4,No : 2, August-2015, PP 420-427.
- [17] A.Nellai Murugan and V.Selvavidhya , Path Related Hetro- Cordial Graphs, International Journal Emerging Technologies in Engineering Research, ISSN 2524-6410,Vol.2, Issue 3 ,October 2015, PP 9-14.
- [18] A.Nellai Murugan and S.Heerajohn, Special Class of Mean Square Cordial Graphs, International Journal Of Applied Research ,ISSN 2394-7500,Vol.1, Issue 11 ,Part B , October 2015, PP 128-131.IF 5.2
- [19] A.Nellai Murugan and J.Shinny Priyanka , Extended Mean Cordial Graphs of Snakes ,International Journal Emerging Technologies in Engineering Research, ISSN 2524-6410,Vol.3, Issue 1 ,October 2015, PP 6-10.
- [20] A.Nellai Murugan and R.Megala Special Class of Relaxed Cordial Graphs ,International Journal Emerging Technologies in Engineering Research, ISSN 2524-6410,Vol.3, Issue 1 ,October 2015, PP 11-14.
- [21] A.Nellai Murugan and P. Iyadurai Selvaraj, Cycle and Armed Cap Cordial graphs, Global Scholastic Research Journal of Multidisciplinary , ISSN 2349-9397,Volume , Issue 11, October 2015, PP 1-14. ISRA 0.416
- [22] A.Nellai Murugan and J.Shinny Priyanka , Path Related Extended Mean Cordial Graphs ,International Journal of Resent Advances in Multi- Disciplinary Research, ISSN 2350-0743,Vol.2, Issue 10 ,October 2015, PP 0836-0840. IF 1.005.
- [23] A.Nellai Murugan and G.Devakiruba., Divisor cordial labeling of Book and  $C_n @ K_1, n.$ , OUTREACH, A Multi-Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 86-92.
- [24] A. Nellai Murugan and P. Iyadurai Selvaraj, Path Related Cap Cordial Graphs, OUTREACH, A Multi- Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 100-106.
- [25] A. Nellai Murugan, and A.Meenakshi Sundari, Product Cordial Graph of Umbrella and  $C_4 @ Sn.$ , OUTREACH , A Multi-Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 113 – 119.
- [26] A. Nellai Murugan, and V.Sripratha, Mean Square Cordial Labeling, OUTREACH, A Multi-Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 125 – 131.
- [27] A.Nellai Murugan and A.L.Poornima, Meanness of Special Class of Graph, OUTREACH, A Multi-Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 140 – 145.
- [28] A.Nellai Murugan and G.Esther, Mean Cordial Labeling of Star, Bi-Star and Wheel, OUTREACH , A Multi-Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 155 – 160.
- [29] A. Nellai Murugan and P. Iyadurai Selvaraj, Cycle & Armed Cap Cordial Graphs, International Journal of Mathematical Combinatorics, ISSN 1937 1055 , Volume II, June 2016,pp 144-152.