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Vague Cosets

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***Abstract*- In this paper we study the vague cosets and their properties.These concepts are used in the development of some important results and theorems about vague groups and vague normalgroups.Also some of their important properties have been investigated.**

***Index Terms*— vague set,vague group, vague normal group, vague coset.**

I. Introduction

The concept proposed by Zadeh.L.A.$[10]$ defining a fuzzy subset A of a given universe X characterizing the membership of an element x of X belonging to A by means of a membership function μ_A defined from X into [0, 1] has revolutionized the theory of Mathematical modeling, Decision making etc., in handling the imprecise real life situations Mathematically. Now, several branches of fuzzy mathematics like fuzzy algebra, Fuzzy topology, fuzzy control theory, fuzzy measure theory etc., have emerged.

But in the decision making, the fuzzy theory takes care of membership of an element x only, that is the evidence of x belonging to A. It does not take care of the evidence of x not belonging to A. It is felt by several decision makers and researchers that in proper decision making, the evidence of x belonging to A and evidence of x not belonging to A are both necessary, and how much x belongs to A or how much x does not belong to A are necessary. Several generalizations of Zadeh's fuzzy set theory have been proposed, one of them such as Vague sets of Gau.W.L.,and Buehrer.D.J [3] (or equivalently Intuituinistic fuzzy sets of Atanasssov.K.T[1] . This concept was applied in several areas like decision making, Fuzzy control, vague carrier decisation making,electro democracy model,knowledge discovery and fault diagnosis etc. It is believed that vague sets will be more useful in decision making,and other areas of mathematical modeling.

A fuzzy set $t_A(x)$ of a set X is a mapping from $X \to [0,1]$, where as a vague set A of set X is a pair (t_A, f_A), where t_A , f_A are functions from $X \to [0,1]$ with $0 \le t_A(x) + f_A(x) \le 1$ for all x in X. Here t_A is called the membership function and f_A is called non-membership function of A. A fuzzy set t_A of X can be identified with the pair $(t_A, 1-t_A)$ .Thus; the theory of vague sets is a generalization of the theory of fuzzy sets. The algebraic aspects of vague sets were initiated by Ranjit Biswas [8] by studying the concepts of vague groups, vague normal groups etc., as generalization of the theory of fuzzy groups etc. Further Ramakrishna [5],[6],[7] continued the study of

vague normal groups, homologous vague groups, vague normalize, vague centralizer, Vague weights and characterizations of cyclic groups in terms of vague groups etc.

In this paper we study the vague cosets, vague symmetry, vague invariant and some of their important properties. Also we proved, if A be a vague group of a group G, hen for all x,y $\in G$

$(1) aA = bA \Leftrightarrow Aa^{-1} = Ab^{-1}$ if A is vague symmetric.

$(2) aA = Aa \Leftrightarrow$ A is vague invariant.

$(3) aA = bA \Leftrightarrow a^{-1}A = b^{-1}A$ if A is vague normal, and

(4) Let A be a vague group of a group G and $a, b \in G$ then

$(i) aA = bA \Leftrightarrow caA = cbA$,

$(ii) aA = bA \Leftrightarrow acA = bcA$

II. Preliminaries

We give here a review of some definitions and results which are in Gau.W.L. and Buehrer D.J[3],Ramakrishna.N[5],Ranjit Biswas [8].

Definition 2.1: A vague set A in the universe of discourse U is a pair (t_A, f_A) where

$t_A : U \to [0,1]$, $f_A : U \to [0,1]$,

are mappings such that $t_A(u) + f_A(u) \le 1$,for all $u \in U$.

The functions t_A and f_A are called true membership function and false membership function respectively.

Definition 2.2: The interval $[t_A(u), 1 - f_A(u)]$ is called the vague value of u in A , and it is denoted by $V_A(u)$. i.e. $V_A(u) = [t_A(u), 1 - f_A(u)]$.

Definition 2.3: Let (G , $*$) be a group. A vague set A of G is called a vague group of G if,for all x,y in G ,

$V_A(xy) \ge min\{V_A(x), V_A(y)\}$ and $V_A(x^{-1}) \ge V_A(x)$.

i.e, $t_A(xy) \ge min\{t_A(x), t_A(y)\}$,

$f_A(xy) \le max\{f_A(x), f_A(y)\}$ and

$t_A(x^{-1}) \ge t_A(x)$, $f_A(x^{-1}) \le f_A(x)$. Here the element x y stands for x $*$ y.

Definition 2.4: A be a vague set of a universe G with true-membership function t_A, and false membership function f_A. For $\alpha, \beta \grave{o} [0,1]$ with $\alpha \le \beta$, the (α, β) cut

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or vague cut of a vague set A is the crisp subset of G is given by $A_{(\alpha,\beta)} = \{x : x\in G , V_A(x) \geq [\alpha,\beta]\}$

i.e, $A_{(\alpha,\beta)} = \{x \mid x\in G, t_A(x) \geq \alpha$,and $1 - f_A(x) \geq \beta\}$.

Definition 2.5: The α -cut, A_α of the vague set A is the (α,α) cut of A ,and hence given by

$A_\alpha = \{x \mid x\in G, t_A(x) \geq \alpha \}$.

Definition 2.6: Let A be a vague group of a group G Then A is called vague normal group if for all x,y $\in$ G , $V_A(xy) = V_A(yx)$.

Alternatively, we can say that, a vague group A is said to be vague normal group of G if $V_A(x) = V_A(yxy^{-1})$ for all x,y $\in$ G .

Notation 2.7: Let I[0,1] denote the family of all closed sub intervals of

[0,1]. If $I_1 = [a_1, b_1]$ and $I_2 = [a_2, b_2]$ be two elements of I[0,1]. We call $I_1 \geq I_2$ if $a_1 \geq a_2$ and $b_1 \geq b_2$ similarly we understand

the relations $I_1 \leq I_2$ and $I_1 = I_2$. Clearly

the relation $I_1 \geq I_2$ does not necessarily imply that $I_1 \supseteq I_2$ and conversely. Also for any two unequal

intervals I_1 and I_2 , there is no necessity that $I_1 \geq I_2$, or

$I_1 \leq I_2$ will be true. The term `imax' means the maximum of two intervals as

imax $\{I_1, I_2\} = [max\{a_1, a_2\}, max\{b_1, b_2\}]$, similarly defined imin. The concept of `imax' and `imin' could be extended to define `isup' and `iinf' of infinite number of elements of I[0,1].

III. VAGUE COSETS

We have the following

Definition 3.1: Let A be a vague group of a group $(G.)$, For any $a \in G,$ a vague left coset of A is denoted by aA and defined by $V_{aA}(x) = V_A(a^{-1}x)$

ie $t_{aA}(x) = t_A(a^{-1}x)$ and $f_{aA}(x) = f_A(a^{-1}x)$

Definition 3.2: Let A be a vague group of a group $(G.)$ For any $a \in G,$ a vague right coset of A is denoted by Aa and defined by

$V_{Aa}(x) = V_A(xa^{-1})$

ie $t_{aA}(x) = t_A(xa^{-1})$ and $f_{Aa}(x) = f_A(xa^{-1})$

Remark 3.3: Clearly vague left coset is a vague set.

Ie $t_{aA}(x) + f_{aA}(x) = t_A(a^{-1}x) + f_A(a^{-1}x) \leq 1$

Similarly in case of vague right coset.

Example 3.4: Consider the group $G = \{1, -1, i, -i\}$ with binary operation as complex multiplication, clearly the set

$A = \{(1, 0.8, 0.1), (-1, 0.7, 0.2), (i, 0.6, 0.2), (-i, 0.5, 0.2)\}$

is a vague set of a group G. Also

$t_{-1A}(1) = t_A((-1)^{-1}.1) = t_A(-1.1) = t_A(-1) = 0.7$

$f_{-1A}(1) = f_A((-1)^{-1}.1) = f_A(-1.1) = f_A(-1) = 0.2$

$t_{-iA}(i) = t_A((-i)^{-1}.i) = t_A(i.i) = t_A(i^2) = 0.7$

$f_{-iA}(1) = f_A((-i)^{-1}.i) = f_A(i.i) = f_A(i^2) = f_A(-1) = 0.2$

$t_{iA}(-i) = t_A((i)^{-1}.i) = t_A(-i.i) = t_A(-i^2) = t_A(1) = 0.7$

$f_{iA}(-i) = f_A((i)^{-1}.i) = f_A(-i.i) = f_A(-i^2) = f_A(1) = 0.2$

Therefore

$-1A = \{(1, 0.7, 0.2), (-1, 0.7, 0.2), (i, 0.7, 0.2), (-i, 0.7, 0.2)\}$

is a vague left coset of A.

Definition 3.5: A vague group A of group G is said to be

(1) vague symmetric if $V_A(x^{-1}) = V_A(x)$ for all $x \in G.$

Ie $t_A(x^{-1}) = t_A(x)$ and $f_A(x^{-1}) = f_A(x)$ for all $x \in G.$

(2) vague invariant if $V_A(xy) = V_A(yx)$ for all $x, y \in G.$

ie $t_A(xy) = t_A(yx)$ and $f_A(xy) = f_A(yx)$ for all $x, y \in G.$

(3) Vague normal if A is both vague symmetry and vague invariant.

Theorem 3.6: Let A be a vague group of a group G. Then for all x,y $\in G$

(1) $aA = bA \Leftrightarrow Aa^{-1} = Ab^{-1}$ if A is vague symmetric.

(2) $aA = Aa \Leftrightarrow$ A is vague invariant.

(3) $aA = bA \Leftrightarrow a^{-1}A = b^{-1}A$ if A is vague normal.

Proof: (1) Suppose A is vague symmetric and aA=bA

$\Leftrightarrow V_{aA}(x) = V_{bA}(x)$

$\Leftrightarrow V_A(a^{-1}x) = V_A(b^{-1}x)$

$\Leftrightarrow V_A[(a^{-1}x))^{-1}] = V_A[(b^{-1}x)^{-1}]$

$\Leftrightarrow V_A(x^{-1}(a^{-1})^{-1}) = V_A(x^{-1}(b^{-1})^{-1})$

$\Leftrightarrow V_{Aa^{-1}}(x^{-1}) = V_{Ab^{-1}}(x^{-1})$

$\Leftrightarrow V_{Aa^{-1}}(x) = V_{Ab^{-1}}(x)$

$(\because A\, is\, vague\, symmetric)$

$ie\; Aa^{-1} = Ab^{-1}$

$aA = bA \Leftrightarrow Aa^{-1} = Ab^{-1}.$

(2) Suppose A is vague invariant

$V_{aA}(x) = V_A(a^{-1}(x)) = V_A(xa^{-1})$

$V_{aA}(x) = V_{Aa}(x)\; for\, all\; x \in G$

$\Rightarrow aA = Aa$

(3) Suppose A is vague normal and $aA = bA$

$\Leftrightarrow Aa^{-1} = Ab^{-1} by$ (1)

$\Leftrightarrow a^{-1}A = b^{-1}A by (2)$

Theorem 3.7: Let A be a vague group of a group G and $a, b \in G$ then

(1) $aA = bA \Leftrightarrow caA = cbA$

(2) $aA = bA \Leftrightarrow acA = bcA$

Proof: Suppose A is vague normal and let $aA = bA$

$\Leftrightarrow V_{aA}[c^{-1}x] = V_{bA}[c^{-1}x] \;\; for \;\; c^{-1}x \in G$

$\Leftrightarrow V_A(a^{-1}c^{-1}x) = V_A(c^{-1}b^{-1}x)$

$\Leftrightarrow V_A((ca)^{-1}x) = V_A((cb)^{-1}x)$

$\Leftrightarrow V_{caA}(x) = V_{cbA}(x)$

$\Leftrightarrow caA = cbA.$

(2)Suppose $aA = bA$

$\Leftrightarrow a^{-1}A = b^{-1}A$

$\Leftrightarrow z^{-1}a^{-1}A = z^{-1}b^{-1}A by (1)$

$\Leftrightarrow (az)^{-1}A = (bz)^{-1}A$

$\Leftrightarrow acA = bcA.$

Now we have the following

Definition 3.8:

Let A be a vague group of a group G, then

$(1)\,{}^{a}A = \{x \in G : xA = aA\}$

$(2) A^a = \{x \in G : Ax = Aa\}$

$(3) aA^e = \{ax \in G : x \in A^e\}$

$(4) A^a A^b = \{xy \in G : x \in A^a \; and \; y \in A^b\}$

Remark 3.9: If A is invariant then we have ${}^{a}A = A^a$ for all $a \in G$

Theorem 3.10: If A be a vague group A of a group G is a vague normal then

(1) For any $a, b \in G, \;\; aA = bA \Leftrightarrow a^{-1}b \in A^e$

$(2) A^e$ is crisp normal subgroup of G .

$(3) A^a = aA^e$ for all $a \in G$

$(4) A^a A^b = A^{ab}$ for all $a, b \in G$

proof: (1) A is vague normal and suppose $aA = bA$

$\Leftrightarrow a^{-1}aA = a^{-1}bA$

$\Leftrightarrow eA = a^{-1}bA$

$\Leftrightarrow a^{-1}bA = A$

$\Leftrightarrow a^{-1}b \in A^e$

$(2) a, b \in A^e$

$\Rightarrow aA = A \; and \; bA = A$

$\Rightarrow aA = bA$

$\Rightarrow a^{-1}aA = a^{-1}bA = A$

$\Rightarrow eA = a^{-1}bA$

$\Rightarrow a^{-1}b \in A^e$

$\Rightarrow A^e$ is a crisp sub group of G.

Let $x \in G, a \in Ae \Rightarrow aA = A$

$\Rightarrow xaA = xA$

$\Rightarrow xax^{-1}A = xx^{-1}A$

$\Rightarrow xax^{-1}A = eA = A$

$\Rightarrow xax^{-1} \in A^e$

thus A^e is a normal subgroup of G.

(3) Take $aA^e = \{ax : x \in A^e\}$

$= \{ax : x \in A^e\}$

$= \{ax : xA = A\}$

$= \{ax : axA = aA\}$

$= \{y : yA = aA\} \quad where \; y = ax \in A$

$= A^a$

$\Rightarrow aA^e = A^a$

(4) Let x, y $\in A^a A^b \Rightarrow x \in A^a \; and \; y \in A^b$

$x \in A^a \Leftrightarrow xA = aA$

$\Leftrightarrow xbA = abA$ and A is vague normal. that is

$x \in A^a \Leftrightarrow xbA = abA$

similarly $y \in A^b \Leftrightarrow xyA = xbA$

implies $xyA = abA \Rightarrow xy \in A^{ab}$

Thus~ $A^a A^b \subseteq A^{ab} ......(i)$

On the other hand,let $x \in A^{ab} \Rightarrow xA = abA$

$\Rightarrow a^{-1}xA = a^{-1}abA$

$\Rightarrow a^{-1}xA = bA \Rightarrow a^{-1}x \in A^b$

i.e $x \in A^{ab} \Rightarrow a^{-1}x \in A^b$

For any

$x \in A^{ab}, x = e.x = aa^{-1}x \, where \, a \in A^a, a^{-1}x \in A^b$

$\Rightarrow aa^{-1}x \in A^a A^b$

$\Rightarrow ex \in A^a A^b \Rightarrow x \in A^a A^b$

$Thus \; A^{ab} \subseteq A^a A^b ......(ii)$

From (i) and (ii) $A^a A^b = A^{ab}$

Definition 3.11: Let G be a group and $x, y \in G.$ we say that x is conjugate to y if there exists $a \in G$ such that $y = a^{-1}xa.$

It is known that the conjugacy is an equivalence relation on G.

The equivalence class in G under the relation of conjugacy is called conjugate class.

Theorem 3.12: Let A be a vague group of a group G.Then A is vague normal iff

A is constant on the conjugate classes of G.

proof: Suppose A is vague normal group of G and $a, b \in G.$

then $V_A(b^{-1}ab) = V_A(abb^{-1}) = V_A(ae) = V_A(a)$

hence A is constant on the conjugate classes of G.

on the other hand A is constant on the conjugate classes of G.

and Let $a, b \in G$ then $V_A(ab) = V_A(ab.aa^{-1})$

$V_A(a(ba)a^{-1}) = V_A(ba)$

Hence A is vague normal group of G.

Theorem 3.13: Let A be vague group of a group G then the following are equivalent.

$(1) aA = Aa \; for each \, a \in G$

$(2) aAa^{-1} = A \; for \, each \;\; a \in G.$

proof: We have A is vague group of a group G and $A \in G.$

Suppose $aA = Aa$

$\Rightarrow aAa^{-1} = Aaa^{-1} = Ae = A$

$\rightarrow aAa^{-1} = A$

On the other hand, suppose $aAa^{-1} = A$

$$\Rightarrow aAa^{-1}a = Aa$$
$$\Rightarrow aAe = Aa$$
$$\Rightarrow aA = Aa.$$

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