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Optimal Imperfect Production Inventory Model with Machine Breakdown and Stochastic Repair Time

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Abstract— **This study develops an imperfect production inventory model with random machine breakdown and stochastic repair time. The model assumes that the production system produces mixture of perfect and imperfect items from the beginning of the production and the system shifts from 'in-control state' to 'out-control state' at random time and also machine repair time is independent of the machine break-down rate. Some portions of the acceptable imperfect products are sale at reduction price and a disposal cost is incurred for environmental protection to dispose the rest portion. Demand of the perfect items is constant and due to imperfect screening, the return items (which are sale as perfect items but originally they are imperfect) are replaced by the new perfect items. Here two cases discussed in comprising with stochastic machine repair time and production down time. MATHEMETICA is used to derive an optimal profit for both the cases. Sensitivity analysis are shown to betterunderst and the model.**

Index Terms— **Imperfect Production, Machine Breakdown, Stochastic repair time, Return items.**

I. Introduction

So far, many EPQ models have been considered by different researchers in different times. In some models, the productions have been discussed on imperfect items. Khouja and Mehrez (1994) assumed the elapsed time until the production process shifts to out-of-control state to be an exponentially with imperfect quality and variable production rate. Salameh and Jaber (2000) developed an inventory model which accounted for imperfect quality items using the EPQ/EOQ formula. More works on imperfect production have been done by Rosenblatt and Lee (1986), Ben-Daya and Hariga (2000), Hayek and Salameh (2001), Goyal and Cardenas-Barron (2002), Chung and Hou(2003), Goyal,Hung and Chen (2003), Sana, Goyal and Chaudhuri(2007), Hazari et.al (2014a, 2014b, 2015) etc.. Manna, Dey and Mondal (2014) presented Three-layer supply chain in an imperfect production inventory model with two storage facilities under fuzzy rough environment. Recently, Manna et.al(2016)develops an imperfect model with promotional demand in random planning horizon using population varying genetic algorithm approach. Many researchers extended the production inventory with machine breakdown model by considering the problems in production such as preventive maintenance,rework etc. The effects of machine breakdown and corrective maintenance on the economic lot sizing were studied by Groenevelt et al.(1992). Abboud (1997)later extended the research of Groenevelt et al.(1992).Giri et al. (2005) developed EMQ model withmachine failure and general repair time. Preventive maintenance is usually used to reduce machine breakdown. In this area, a lot of research works was done by Cheung and Hausman (1997), Dohi et al. (2001),Lin and Gong (2006), Lo et al. (2007), El-Ferik(2008), Liaet al. (2009), Chiu et al. (2010), Chiu (2010), Chiu and Chang (2014) etc. From the authors' literature search, very few researchers have considered imperfect production inventory model with return items under stochastic machine breakdown and stochastic repair time.Lin and Gong (2006)developed EPQ deteriorating inventory model with machine breakdown and fix repair time. Widyadana and Wee (2011) extend this idea with stochastic repair time. This paper extend the research of Widyadana and Wee (2011) with imperfect production in which the production system shifts from 'in-control state' to 'out-control state' at random time and simultaneously, return items from the markets are consider and replace them by new perfect items. Here two cases are discuss in comprising with stochastic machine repair time and production down time.

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II. MATHEMATICAL FORMULATION OF PROSOSED INVENTORY MODEL

The following assumptions and notations are adopted for this model.

A. *Assumptions*

1. The production rate is constant.

2. At the beginning of the production process, the process may shift from 'in-control state' to 'out of control state' and produces imperfect quality items and follows a probability density function $f(t) = \alpha e^{-\alpha t}, \alpha > 0, t > 0$

3. Full inspection process is consider. Only the $\beta\ (0 < \beta < 1)$times of total imperfect quality items are sale at a low price rate(reduction sale). and $(1-\beta)$ times of imperfect quality items are rejected with a certain cost.

4. d_1(constant) be the demand rate of perfect quality items throughout the production cycle.

5.w($=d_1$ y) be the returned rate items from the customer and meet by replacing the new product (0<y<0.1)

6..The demand rate, $d_2=d_3+d_4\ e^{\frac{u}{v}}$, where u= $s_{max} - s_1$ *and* $v = s_1 - s_{\min}$ is for acceptable imperfect quality items. whered_3 , d_4 both are constant.

7. Immediately after the end of production up time, reduction sale of acceptable imperfect items are consider.

8. The time horizon is infinite.

9. Machine repair time is stochastic and independent of machine break down.

B. Notations

P : The production rate is constant.

d_1 : Demand rate of perfect quality items.

d_2: Demand rate of acceptable imperfect quality items.

I_p: Inventory of the perfect quality items at time t.

I_d : Inventory of the imperfect quality items at time t.

Q : Total Inventory in the production cycle.

w :Rate of return items from the customer.

T_1 :Time of production stop.

T_p : Time when machine breakdown occur.

T_2 :Time of perfect quality items is exhaust.

T_3 :Time of imperfect quality items is exhaust.

c_h : Holding cost per item per unit time.

c_p :Production cost per unit item.

c_d : :Disposal cost per unit item.

s : Price rate of perfect quality items.

s_1 : Price rate(reduced) of acceptable imperfect quality items.

E(Q) : Expected inventory holding in the production cycle.

E(HC) : Expected inventory holding cost.

E(P): Expected production quantity in the production cycle.

E(PC) : Expected production cost.

E(T) : Expected duration of the production cycle.

E(CLS) : Expected lost sale cost.

E(DC) : Expected disposal cost.

E(TC) : Expected total cost.

E(SR) : Expected sales revenue.

E(APC1) : Expected average profit when lost sale occurs.

E(APC2) : Expected average profit when lost sale does not occurs.

C. Mathematical Formulation

Here, we consider a manufacturing process , in which the system produces both perfect and imperfect quality items from the beginning of the production run.

The perfect and imperfect quality items are separates by a inspection section. This section select the acceptable quality items, β times of the total imperfect quality items which are suitable for the low price rate(i.e the reduction sale) and (1- β) times of imperfect quality items are rejected(scrapped) with environmental conciseness with a certain cost.

The probability of imperfect quality of total production follow the probability density function $f(t) = \alpha e^{-\alpha t}, \alpha > 0, t > 0$. If P be the production rate, theexpected quantity of imperfect products at time t is $(1 - e^{-\alpha t})$. Therefore the perfect quality item is $Pe^{-\alpha t}$.

Let d1be the demand of perfect quality items and w=d1 y , where y is the proportion of the imperfect (defective) items of the sold perfect quality items, returned from the customer and which can be meet by replacing the new product of perfect quality items. Here (0<y<0.1) ..The acceptable imperfect quality items are sale immediately after the production up-time with a demand ,d2=d3+d4 $e^{\frac{u}{v}}$, where u=$s_{max} - s_1$ *and* $v = s_1 - s_{\min}$ where s1 is the reduced price rate.

At the beginning of the production run the process are shift to out of control state from in control state and produces imperfect items. For the production cycle, Ip(t) denotes the inventory level of perfect quality items at time t . The system of perfect production starts at t=0 with rate $Pe^{-\alpha t}$ and the inventory of perfect items increases at a rate $Pe^{-\alpha t} - (d_1 + w)$ until time t=T1and at the same time Id(t) denotes the inventory level of imperfect quality items at time t, and the inventory of imperfect items increases at a rate $P(1 - e^{-\alpha t})$until time t=T1, t=T2 and t=T3denotes the production downtime of perfect and imperfect quality items and inventory decline with demand rates (d1+w) and d2 simultaneously.

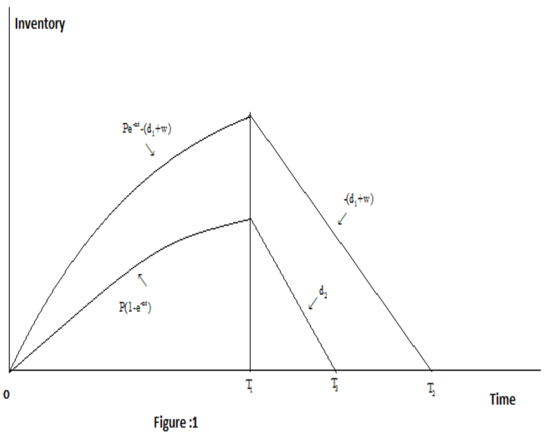

Fig-1: The Behavior of Such a Unified System is Depicted

III. THE INVENTORY LEVELS ARE GOVERNED BY THE FOLLOWING DIFFERENTIAL EQUATIONS

A. Perfect Quality Inventory

The governing differential equation of the perfect quality item is

$$\frac{dI_p(t)}{dt} = \begin{cases} Pe^{-\alpha t} - (d_1 + w); 0 \le t \le T_1 \\ -(d_1 + w) \qquad ; T_1 \le t \le T_2 \end{cases} \tag{1}$$

with$I_p(0)=0$ and $I_p(T_2)=0$.

B. Imperfect Quality Inventory

The governing differential equation of the imperfect quality item is

$$\frac{dI_d(t)}{dt} = \begin{cases} \beta P(1 - e^{-\alpha t}); 0 \le t \le T_1 \\ -d_2 \qquad ; T_1 \le t \le T_3 \end{cases} \tag{2}$$

with $I_d(0)=0$ and $I_d(T_3)=0$. Where $\beta(0 < \beta < 1)$ is the portion of imperfect items produced during the production run are consider for reduction sale with demand $d_2=d_3+d_4e^{\frac{u}{v}}$, where $u=s_{max}-s_1$ and $v=s_1-s_{min}$, increases exponentially with low price rate.

From (1) and (2), it follows

$$I_p(t)=\begin{cases}\frac{P}{\alpha}(1-e^{-\alpha t})-(d_1+w)t; 0\le t\le T_1\\ \frac{P}{\alpha}(1-e^{-\alpha T_1})-(d_1+w)t; T_1\le t\le T_2\end{cases} \quad (3)$$

$$I_d(t)=\begin{cases}\frac{\beta P}{\alpha}(-1+\alpha t+e^{-\alpha t}) & ;0\le t\le T_1\\ \frac{\beta P}{\alpha}(-1+\alpha T_1+e^{-\alpha T_1})-d_2(t-T_1); T_1\le t\le T_3\end{cases} \quad (4)$$

From the continuity condition, we get

$$T_2=X(1-e^{-\alpha T_1}) \quad \text{, where } X=\frac{P}{\alpha(d_1+w)} \quad (5)$$

$$T_3=\frac{\beta P}{\alpha}(-1+\alpha T_1+e^{-\alpha T_1})+T_1 \quad (6)$$

For feasibility, $T_2>T_1$, which gives

$$\frac{P}{\alpha}(1-e^{-\alpha T_1})>(d_1+w)T_1 \quad (7)$$

Inventory for the perfect quality items is

$$Q_p=\int_0^{T_1}[\frac{P}{\alpha}(1-e^{-\alpha t})-(d_1+w)t]dt$$
$$+\int_{T_1}^{T_2}[\frac{P}{\alpha}(1-e^{-\alpha T_1})-(d_1+w)t]dt$$
$$=\frac{P(P-2(d_1+w))}{2\alpha^2(d_1+w)}+\frac{P}{\alpha}T_1e^{-\alpha T_1} \quad (8)$$
$$+\frac{P(-P+(d_1+w))}{\alpha^2(d_1+w)}e^{-\alpha T_1}+\frac{P^2}{2\alpha^2(d_1+w)}e^{-2\alpha T_1}$$

Inventory for the imperfect quality items is

$$Q_d=\int_0^{T_1}[\frac{\beta P}{\alpha}(-1+\alpha t+e^{-\alpha t})\,]dt+$$
$$\int_{T_1}^{T_2}[\frac{\beta P}{\alpha}(-1+\alpha T_1+e^{-\alpha T_1})-d_2(t-T_1)]dt$$
$$=(\frac{\beta P}{\alpha^2}+\frac{\beta^2P^2}{2\alpha^2 d_2})+(\frac{\beta P}{2}+\frac{\beta^2P^2}{2d_2})T_1^2 \quad (9)$$
$$-(\frac{\beta P}{\alpha}+\frac{\beta^2P^2}{\alpha d_2})T_1-(\frac{\beta P}{\alpha^2}+\frac{\beta^2P^2}{\alpha^2 d_2})e^{-\alpha T_1}$$
$$+\frac{\beta^2P^2}{\alpha d_2}T_1e^{-\alpha T_1}+\frac{\beta^2P^2}{2\alpha^2 d_2}e^{-2\alpha T_1}$$

Hence the total inventory is

$$Q=Q_p+Q_d$$
$$=A+BT_1+CT_1^2+De^{-\alpha T_1}+ET_1e^{-\alpha T_1}+Fe^{-2\alpha T_1} \quad (10)$$

Where ,

$$A=\frac{P^2}{2\alpha^2}(\frac{1}{(d_1+w)}+\frac{\beta^2}{d_2})+\frac{P}{\alpha^2}(1+\beta)$$
$$B=-\frac{\beta P}{\alpha}(1+\frac{\beta P}{d_2}) \quad , C=\frac{\beta P}{2}(1+\frac{\beta P}{d_2}) \quad -$$
$$D=-\frac{P^2}{\alpha^2}(\frac{1}{(d_1+w)}+\frac{\beta^2}{d_2})+\frac{P}{\alpha^2}(1-\beta)$$
$$E=\frac{P}{\alpha}(1+\frac{\beta^2P}{d_2}) \quad , F=\frac{P^2}{2\alpha^2}(\frac{1}{(d_1+w)}+\frac{\beta^2}{d_2}) \quad (11)$$

Case 1: Machine Breakdown And Stochastic Machine Repair Time Exceed Production Down Time

If the machine breakdown occurs at the time point $t=T_p$ and follow the probability density function $f(T_p)=\delta e^{-\delta T_p}, \delta>0 \text{ and } T_p>0$. Then

$$Q=\begin{cases}A+BT_p+CT_p^{\,2}+De^{-\alpha T_p}\\ +ET_pe^{-\alpha T_p}+Fe^{-2\alpha T_p} & ;T_p\le T_1\\ A+BT_1+CT_1^2+De^{-\alpha T_1}\\ +ET_1e^{-\alpha T_1}+Fe^{-2\alpha T_1} & ;T_p\ge T_1\end{cases} \quad (12)$$

The Expected inventory holding is

$$E(Q)=\int_0^{T_1}[A+BT_p+CT_p^{\,2}+De^{-\alpha T_p}+ET_pe^{-\alpha T_p}$$
$$+Fe^{-2\alpha T_p}]\delta e^{-\delta T_p}dT_p+\int_{T_1}^{\infty}[A+BT_1+CT_1^2+De^{-\alpha T_1}$$
$$+ET_1e^{-\alpha T_1}+Fe^{-2\alpha T_1}]\delta e^{-\delta T_p}dT_p$$

Hence expected inventory holding cost is

$$E(HC)=c_h[\frac{A\delta^2+B\delta+2C}{\delta^2}+\frac{F\delta}{(2\alpha+\delta)}+$$
$$\frac{\delta(D(\delta+\alpha)+E)}{(\alpha+\delta)^2}-\frac{B\delta+2C}{\delta^2}e^{-\delta T_1}-\frac{2C}{\delta}T_1e^{-\delta T_1} \quad (13)$$
$$+\frac{D\alpha(\alpha+\delta)-E\delta}{(\alpha+\delta)^2}e^{-(\alpha+\delta)T_1}+\frac{\alpha E}{\alpha+\delta}T_1e^{-(\alpha+\delta)T_1}$$
$$+\frac{2\alpha E}{2\alpha+\delta}e^{-(2\alpha+\delta)T_1}]$$

Expected production in production run

$$E(P)=\int_0^{T_1}PT_p\delta e^{-\delta T_p}dT_p+\int_{T_1}^{T_2}PT_1\delta e^{-\delta T_p}dT_p$$

is $\quad (14)$

$$=\frac{P}{\delta}(1-e^{-\delta T_1})$$

Expected production cost is

$$E(PC)=c_p\frac{P}{\delta}(1-e^{-\delta T_1}) \quad (15)$$

C. Machine Repair Time is Stochastic

When the machine breakdown occur with in the production run time, the machine repair time is stochastic which is uniformly distributed in (o,μ) with density function

$$g(t)=\begin{cases}\dfrac{1}{\mu} & ;0\le t\le \mu \\ 0 & ;elsewhere\end{cases} \qquad (16)$$

Therefore Expected duration of cycle is

$$E(T)=\int_0^{T_1}T_2\delta e^{-\delta T_p}dT_p+\int_{T_1}^{\infty}T_2\delta e^{-\delta T_p}dT_p+\int_0^{T_1}\int_{T_1}^{T_2}(t-T_2)g(t)\delta e^{-\delta T_p}dT_p \qquad (17)$$

Here T_2 represent the end of the production cycle and3rd term in the above expression represent the extra time for machine repair.

$$E(T_2)=\frac{X\alpha}{\alpha+\delta}(1-e^{-(\alpha+\delta)T_1}) \qquad (18)$$

Again expected extra time for machine repair is

$$E(ET)=\int_0^{T_1}\int_{T_1}^{T_2}(t-T_2)g(t)\delta e^{-\delta T_p}dT_p$$

$$=\frac{\delta}{2\mu}[\frac{(\mu-X)^2}{\delta}(1-e^{-\delta T_1})+\frac{2\mu X-2X^2}{\delta+\alpha}(1-e^{-(\alpha+\delta)T_1})+\frac{X^2}{\delta+2\alpha}(1-e^{-(2\alpha+\delta)T_1})] \qquad (19)$$

From above, it follows that

$$E(T)=\frac{X\mu(\alpha+\delta)-\delta X^2}{\mu(\alpha+\delta)}(1-e^{-(\alpha+\delta)T_1})+\frac{(\mu-X)^2}{2\mu}(1-e^{-\delta T_1})+(1-e^{-(2\alpha+\delta)T_1})\frac{\delta X^2}{2\mu(2\alpha+\delta)} \qquad (20)$$

Expected lost sale cost is

$$E(CLS)=d_1s\frac{\delta}{2\mu}[\frac{(\mu-X)^2}{\delta}(1-e^{-\delta T_1})+\frac{2\mu X-2X^2}{\delta+\alpha}(1-e^{-(\alpha+\delta)T_1})+\frac{X^2}{\delta+2\alpha}(1-e^{-(2\alpha+\delta)T_1})] \qquad (21)$$

Expected disposal amount of imperfect product in the production process is

$$E(IMP)=(1-\beta)P[\frac{1}{(\alpha+\delta)}e^{-(\alpha+\delta)T_1}-\frac{1}{\delta}e^{-\delta T_1}+\frac{\alpha}{\delta(\alpha+\delta)}] \qquad (22)$$

Expected disposal cost for environmental point of view is

$$E(DC)=c_d(1-\beta)P[\frac{1}{(\alpha+\delta)}e^{-(\alpha+\delta)T_1}-\frac{1}{\delta}e^{-\delta T_1}+\frac{\alpha}{\delta(\alpha+\delta)}] \qquad (23)$$

Expected total cost = E(TC)

= Setup cost (including inspection cost) k + Expected Production cost (E(PC)) + Expected inventory holding cost(E(HC)) + Expected lost sale cost(E(CLS)) + Expected disposal cost(E(DC))

$$E(TC)=K+c_h[\frac{A\delta^2+B\delta+2C}{\delta^2}+\frac{F\delta}{(2\alpha+\delta)}+\frac{\delta(D(\delta+\alpha)+E)}{(\alpha+\delta)^2}-\frac{B\delta+2C}{\delta^2}e^{-\delta T_1}-\frac{2C}{\delta}T_1e^{-\delta T_1}+\frac{D\alpha(\alpha+\delta)-E\delta}{(\alpha+\delta)^2}e^{-(\alpha+\delta)T_1}+\frac{\alpha E}{\alpha+\delta}T_1e^{-(\alpha+\delta)T_1}+\frac{2\alpha E}{2\alpha+\delta}e^{-(2\alpha+\delta)T_1}]+c_p\frac{P}{\delta}(1-e^{-\delta T_1})+d_1s\frac{\delta}{2\mu}[\frac{(\mu-X)^2}{\delta}(1-e^{-\delta T_1})+\frac{2\mu X-2X^2}{\delta+\alpha}(1-e^{-(\alpha+\delta)T_1})+\frac{X^2}{\delta+2\alpha}(1-e^{-(2\alpha+\delta)T_1})]+c_d(1-\beta)P[\frac{1}{(\alpha+\delta)}e^{-(\alpha+\delta)T_1}-\frac{1}{\delta}e^{-\delta T_1}+\frac{\alpha}{\delta(\alpha+\delta)}] \qquad (24)$$

2.5.2 Sales revenue

We consider s_1 be the price rate of acceptable imperfect quality items, and then sales revenue in the complete cycle is

$$sd_1X(1-e^{-\alpha T_1})+s_1\beta P(\frac{e^{-\alpha T_1}+\alpha T_1-1}{\alpha}) \qquad (25)$$

If the machine breakdown occurs at t=T_p , then expected sales revenue from the complete production cycle is

$$E(SR)=$$

$$\int_0^{T_1}[sd_1X(1-e^{-\alpha T_p})+s_1\beta P(\frac{e^{-\alpha T_p}+\alpha T_p-1}{\alpha})]\delta e^{-\delta T_p}dT_p+\int_{T_1}^{\infty}[sd_1X(1-e^{-\alpha T_1})+s_1\beta P(\frac{e^{-\alpha T_1}+\alpha T_1-1}{\alpha})]\delta e^{-\delta T_p}dT_p$$

$$=\frac{d_1Xs\alpha-P\beta s_1}{\alpha+\delta}(1-e^{-(\alpha+\delta)T_1})+\frac{P\beta s_1}{\delta}(1-e^{-\delta T_1})$$

(26) Expected average profit per unit time= (Expected sales revenue-Expected total cost)/Expected production cycle

$$E(APC1)=\{[\frac{d_1Xs\alpha-P\beta s_1}{\alpha+\delta}(1-e^{-(\alpha+\delta)T_1})+\frac{P\beta s_1}{\delta}(1-e^{-\delta T_1})]-K+c_h[\frac{A\delta^2+B\delta+2C}{\delta^2}+\frac{F\delta}{(2\alpha+\delta)}+\frac{\delta(D(\delta+\alpha)+E)}{(\alpha+\delta)^2}-\frac{B\delta+2C}{\delta^2}e^{-\delta T_1}-\frac{2C}{\delta}T_1e^{-\delta T_1}+\frac{D\alpha(\alpha+\delta)-E\delta}{(\alpha+\delta)^2}e^{-(\alpha+\delta)T_1}+\frac{\alpha E}{\alpha+\delta}T_1e^{-(\alpha+\delta)T_1}+\frac{2\alpha E}{2\alpha+\delta}e^{-(2\alpha+\delta)T_1}]+c_p\frac{P}{\delta}(1-e^{-\delta T_1})+d_1s\frac{\delta}{2\mu}[\frac{(\mu-X)^2}{\delta}(1-e^{-\delta T_1})+\frac{2\mu X-2X^2}{\delta+\alpha}(1-e^{-(\alpha+\delta)T_1})+\frac{X^2}{\delta+2\alpha}(1-e^{-(2\alpha+\delta)T_1})]+c_d(1-\beta)P[\frac{1}{(\alpha+\delta)}e^{-(\alpha+\delta)T_1}-\frac{1}{\delta}e^{-\delta T_1}+\frac{\alpha}{\delta(\alpha+\delta)}]\}/\{\frac{X\mu(\alpha+\delta)-\delta X^2}{\mu(\alpha+\delta)}(1-e^{-(\alpha+\delta)T_1})+\frac{(\mu-X)^2}{2\mu}(1-e^{-\delta T_1})+(1-e^{-(2\alpha+\delta)T_1})\frac{\delta X^2}{2\mu(2\alpha+\delta)}\} \qquad (27)$$

With $\frac{P}{\alpha}(1-e^{-\alpha T_1}) > (d_1+w)T_1$ (28)

And condition of lost sale is

$$P(1-e^{-\alpha T_1}) < \alpha(d_1+w)\mu \tag{29}$$

Case 2: Machine Breakdown And Stochastic Machine Repair Time Not Exceed Production Down Time

If the machine breakdown occur and machine repair time is stochastic but less than production down time T2 . In this case lost sales does not occur and so,

Expected total cost=E(TC)

= Setup cost(including inspection cost) (K) + Expected Production cost (E(PC)) + Expected inventory holding cost (E(HC)) + Expected disposal cost (E(DC)).

$$E(TC) = K + c_h[\frac{A\delta^2+B\delta+2C}{\delta^2} + \frac{F\delta}{(2\alpha+\delta)} + \frac{\delta(D(\delta+\alpha)+E)}{(\alpha+\delta)^2} - \frac{B\delta+2C}{\delta^2}e^{-\delta T_1} - \frac{2C}{\delta}T_1e^{-\delta T_1} + \frac{D\alpha(\alpha+\delta)-E\delta}{(\alpha+\delta)^2}e^{-(\alpha+\delta)T_1} + \frac{\alpha E}{\alpha+\delta}T_1e^{-(\alpha+\delta)T_1} + \frac{2\alpha E}{2\alpha+\delta}e^{-(2\alpha+\delta)T_1}] + c_p\frac{P}{\delta}(1-e^{-\delta T_1}) + c_d(1-\beta)P[\frac{1}{(\alpha+\delta)}e^{-(\alpha+\delta)T_1} - \frac{1}{\delta}e^{-\delta T_1} + \frac{\alpha}{\delta(\alpha+\delta)}] \tag{30}$$

and Expected sales revenue for complete cycle is

$$E(SR) = \int_0^{T_1}[sd_1X(1-e^{-\alpha T_p}) + s_1\beta P(\frac{e^{-\alpha T_p}+\alpha T_p-1}{\alpha})]\delta e^{-\delta T_p}dT_p + \int_{T_1}^{\infty_1}[sd_1X(1-e^{-\alpha T_1}) + s_1\beta P(\frac{e^{-\alpha T_1}+\alpha T_1-1}{\alpha})]\delta e^{-\delta T_p}dT_p \tag{31}$$

$$= \frac{d_1Xs\alpha - P\beta s_1}{\alpha+\delta}(1-e^{-(\alpha+\delta)T_1}) + \frac{P\beta s_1}{\delta}(1-e^{-\delta T_1})$$

Expected production cycle is

$$E(T_2) = \frac{X\alpha}{\alpha+\delta}[1-e^{-(\alpha+\delta)T_1}] \tag{32}$$

In this case ,Expected average profit per unit time= (Expected sales revenue-Expected total cost)/Expected production cycle.

$$E(APC2) = \{[\frac{d_1Xs\alpha - P\beta s_1}{\alpha+\delta}(1-e^{-(\alpha+\delta)T_1}) + \frac{P\beta s_1}{\delta}(1-e^{-\delta T_1})] - [K + c_h[\frac{A\delta^2+B\delta+2C}{\delta^2} + \frac{F\delta}{(2\alpha+\delta)} + \frac{\delta(D(\delta+\alpha)+E)}{(\alpha+\delta)^2} - \frac{B\delta+2C}{\delta^2}e^{-\delta T_1} - \frac{2C}{\delta}T_1e^{-\delta T_1} + \frac{D\alpha(\alpha+\delta)-E\delta}{(\alpha+\delta)^2}e^{-(\alpha+\delta)T_1} + \frac{\alpha E}{\alpha+\delta}T_1e^{-(\alpha+\delta)T_1} + \frac{2\alpha E}{2\alpha+\delta}e^{-(2\alpha+\delta)T_1}] + c_p\frac{P}{\delta}(1-e^{-\delta T_1}) + c_d(1-\beta)P[\frac{1}{(\alpha+\delta)}e^{-(\alpha+\delta)T_1} - \frac{1}{\delta}e^{-\delta T_1} + \frac{\alpha}{\delta(\alpha+\delta)}]]\} / \{\frac{X\alpha}{(\alpha+\delta)}(1-e^{-(\alpha+\delta)T_1})\} \tag{33}$$

IV. Solution

Our objective is to find the optimal value of T1 for which in either case the objective function E(APC1) and E(APC2) are maximum. So that the necessary condition for objective function to be maximized is

Production rate	Optimal Run time	Production Cycle of Imperfect items	Production Cycle of Imperfect items	Profit
600	5.0776	7.3583	9.6848	38109.0
550	5.4636	7.8585	9.4010	40629.0
500	5.8794	8.3718	9.0496	43966.8
450	6.3403	8.6105	8.9163	48188.0

$\frac{dE(APC1)}{dT_1} = 0$ and $\frac{d^2E(APC1)}{dT_1^2} < 0$ along with the condition (28) and (29) and $\frac{dE(APC1)}{dT_1} = 0$ and $\frac{d^2E(APC1)}{dT_1^2} < 0$ along with the condition (28) . Using MATHEMATICA , we see that Average profit function is concave for T1>0.

V. Numerical Examples

The parametric values in the model are as P=500;d1=250; d3=200;d4=4 ; α =0.09; β =0.8; δ =0.1 ; μ =2; ch=10;cp =100;s=1000; ,smax=500;smin=300;y=0.01;w=y*d1;k=500;cd=2.We analyses the effect on optimum production runtime ,optimum production cycle for both imperfect item and perfect item and on optimum profit for different selling price of imperfect items.

Table-1: Selling price of imperfect items And Profit.

Selling Price of Imperfect Items	Optimal Run time	Production Cycle of Imperfect items	Production Cycle of Imperfect items	Profit
400	5.87944	8.37182	9.04957	43066.8

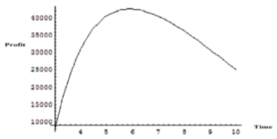

Fig 2: (Total profit per unit time Vs Time for all different parameter)

Table 2: For Different Selling Price of Imperfect Items

Selling Price of Imperfect Items	Optimal Run time	Production Cycle of Imperfect items	Production Cycle of Imperfect items	Profit
50	5.91079	8.49244	9.0771	46242.0
420	5.88912	8.42615	0.0518	44899.6
400	5.87944	8.37182	9.0495	43066.8
370	5.89177	8.23023	9.0549	42346.4
350	5.98447	7.92124	9.1625	40573.3

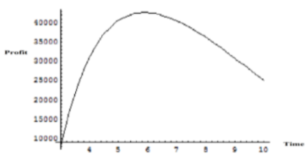

Fig 2: Total profit per unit time Vs Time for all different parameter

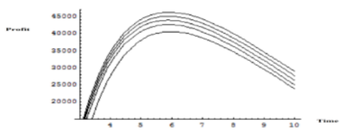

Fig 3: (Total profit per unit time for different selling price of imperfect items)

VI. For different production rate of items

Table-3: Effect for different production rate of items.

Production rate	Optimal Run time	Production Cycle of Imperfect items	Production Cycle of Imperfect items	Profit
600	5.0776	7.3583	9.6848	38109.0
550	5.4636	7.8585	9.4010	40629.0
500	5.8794	8.3718	9.0496	43966.8
450	6.3403	8.6105	8.9163	48188.0

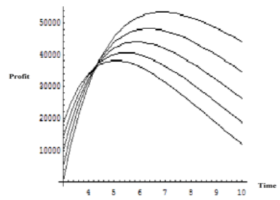

Fig -4: Total Profit per Unit Time for Different Production Rate of Items

VII. For different return rate of perfect items from market

Table-4: Effect for different Return rate of perfect items from market.

Different Value of y	Optimal Run time	Production Cycle of Imperfect items	Production Cycle of Imperfect items	Profit
0.01	5.87943	8.37182	9.04057	43966.8
0.03	6.04282	8.66389	9.05055	41938.5
0.05	6.20789	8.96166	9.05934	39956.0
0.07	6.37467	9.26521	9.066979	38017.3
0.09	6.54319	9.57464	.07351	36120.0

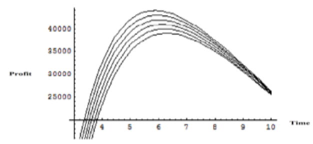

Fig 5: (Total profit per unit time for different return rate of perfect items from market.)

VIII. Conclusion

In this study, an imperfect production inventory model with random machine breakdown and stochastic repair time is developed where machine repair time is independent of the machine break-down rate. Stochastic repair time is distributed uniformly and the stochastic time of machine breakdown is exponentially distributed. Demand for perfect quality items is constant and the demand of the acceptable quality of imperfect items are highly sensitive with selling price. Due to imperfect screening, the imperfect items which are sale as perfect items are return back from the market are replaced by the new perfect items. Here two cases discussed in comprising with stochastic machine repair time and production down time. MATHEMATICA is used to derive an optimal profitfor both the cases. We analyze the effect on optimum production runtime ,optimum production cycle for both imperfect item and perfect item and on optimum profit for different selling price of imperfect items.

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