The Effects of Warm Plasma and HF Electrical Field on Beam-Plasma Interaction in Plasma Waveguide

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Abstract—We investigated in this paper the stabilizing effects of warm plasma and a strong HF electrical field on a two-stream (Buneman) instability in a plane and cylindrical plasma waveguide. Starting from the two-fluid plasma model to separate the problem into two parts, temporal and space. Plasma electrons considered to have a thermal velocity. It is shown that independent of the geometry of the problem the conclusion on the HF stabilization of Buneman instability in a plasma waveguide remains valid and the growth rate of the instability in warm plasma has been reduced compared to that of cold plasma.

The influence of HF electric field on the instability of a low-density electron beam passing through warm plasma waveguide is also reported.

Index Terms—Separation method, Beam-plasma interaction, Buneman instability, Nonuniform Plasma, Warm plasma waveguide.

I. INTRODUCTION

It has been shown [1] that dispersion equations describing parametric excitation of surface waves at the boundary of isotropic plasma-vacuum to within the eigen frequency renormalization coincide with the equations that determine the Parametric excitation of volumetric waves in uniform unbounded plasma. Proceeding from this conclusion the method for investigation of parametric interaction of external HF electrical field with electrostatic oscillations in isotropic bounded nonuniform plasma has been proposed [2-4]. The method makes it possible to separate the problem into two parts. The “dynamical” (temporal) part describes the parametric build up of oscillations and corresponding equations within the renormalization of natural (eigen) frequencies coincide with equations for parametrically unstable waves in an infinite uniform plasma [2,4]. Natural frequencies of oscillations and spatial distribution of the amplitude of the self-consistent electric field are determined from the solution of a boundary-value problem (“spatial” part) taking into account specific spatial distribution of plasma density. The proposed approach (“separation method”) is significantly simpler than the method previously used in the theory of parametric resonance in nonuniform plasma [1,5,6,7].

The control of instabilities, in particular their suppression, could be achieved by using intense high-frequency (HF) fields [8-15]. Based on existing experimental and theoretical results, it may be deduced that the effect of an intense electromagnetic radiation on plasma may produce a qualitative change in its main properties [5,16-19]. In particular, the plasma dispersion characteristics can change to some extent: the absorption of an external (pump) electromagnetic field energy by plasma particles increases in comparison with collisional absorption, the possibility exists for parametric excitation of plasma waves, and for stabilization of many plasma instabilities.

The stabilization effect of a uniform HF electric field on a two-stream (Buneman) instability in uniform unbounded plasma has been for the first time investigated in [19]. The dispersion equation for characteristic frequencies of electrostatic oscillations excited by relative motion of electrons and ions in a HF electric field has been obtained and analyzed. The presence of a pump wave strongly modifies the dispersion equation of Buneman instability. As a consequence the growth rate of instability reduces in comparison with the growth rate at vanishing external field amplitude.

The extremely interesting properties of plasma located in a strong high frequency (HF) electric field have stimulated a broad range of theoretical investigations in this promising field of physics.

The present paper is concerned with the investigation of the influence of an intense HF electric field on instabilities excited by electron beam in a plane and cylindrical warm plasma waveguide. In section two we describe the modified version of the method used before [3] in the problem of parametric interaction of HF field with electrostatic surface waves in an isotropic bounded warm plasma. The method makes it possible to separate the problem into two parts. The “dynamical” (temporal) part enables the frequencies and growth rates of unstable oscillations to be found and corresponding equations within

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ISSN: 2455-3956, Volume-2, Issue-5, May 2016 Pages 83-88
the renormalization of in an infinite uniform plasmanatural (eigen) frequencies coincides with equations describing beam-plasma interaction in an unbounded warm plasma immersed into HF electric field. Natural frequencies of oscillations and spatial distribution of the self-consistent electrical field amplitude are determined from the solution of a boundary-value problem (“spatial” part) taking into account specific spatial distribution of plasma density. The method described is used for solution of two problems.

In section three the effect of HF electric field on the excitation of surface waves by an electron beam under the development of Buneman instability is considered. It is shown that in a bounded warm plasma application of HF field decrease the region of instability and reduces the growth rate of unstable oscillations. This result validates the conclusions of ref. [19] on HF stabilization of two-stream instability in unbounded warm plasma.

The instability of a low-density electron beam passing through warm plasma waveguide in the presence of HF electric field is analyzed in section four. In contrast to the case of Buneman instability, HF field has no essential influence on dispersion characteristics of unstable surface waves excited in a warm plasma waveguide by a low-density electron beam. The region of instability only slightly narrowing and the growth rate decreases by a small parameter and this result has been reduced compared to cold plasma. Also, it is found that the plasma electrons have not affected the solution of the space part of the problem.

II. SEPARATION METHOD IN THE PROBLEM OF A BEAM-PLASMA INTERACTION IN BOUNDED WARM PLASMA UNDER THE EFFECT OF HF ELECTRIC FIELD.

We assume that a uniform cold electron beam propagating along radially nonuniform cylindrical warm plasma waveguide. The radius of the beam $R$ is supposed to coincides with the radius of the plasma cylinder. We take the vector of the external HF field $E_p = E_0 \sin(\omega_b t)$ to be oriented along the axis of the plasma cylinder. Equilibrium density of beam and plasma $n_{a_0}$ varies radially.

“Separation method” has been described [6,20] in application to the problem of parametric excitation of surface waves in a cold isotropic plasma. Here we shall follow this paper. Representing the perturbations of velocity, density and electrical potential in the form

$$\delta V_{\alpha}, \delta n_{\alpha}, \Phi \sim \exp i (m_1 \Psi + k z)$$

the linearized set of hydrodynamical equations together with Poisson equation can be reduced to the form

$$\frac{\partial^2 V_{\alpha}}{\partial t^2} + \eta V_{\alpha} = -\frac{m_e p^2}{e^2} e^{i \omega_0} \sum_{\beta=0, \pm 1} \frac{e_\beta^2}{m_\beta} e^{-i \nu_\beta},$$

where $\eta = k^2 V_{\alpha_0}^2$, $\alpha = e, i$ and $p$ is a separation constant, and $V_{\alpha_0}$ is the temporal part of the density perturbations ($V_{\alpha} = V_{\alpha_0} (t) \nu_{\alpha} (r)$),

$$A_\alpha = (ku_{\alpha_0}) t - a_\alpha \sin (\omega_0 t), \text{ and } a_\alpha = \frac{e_\alpha k E_0}{m_{\alpha_0} \omega_0^2} \approx a_e. \quad (2)$$

Assuming that the ions are at rest ($u_{b_0} \equiv 0$) and that the frequency of the HF field is much larger than the eigen frequencies of the excited surface waves ($\omega_0 >> \omega_{sw} \approx \omega_p$), we may use the method of averaging.

Introducing the explicit form of $A_\alpha (a_b = a_e >> a_i)$, and a new variable $W_\alpha = \frac{m_\alpha}{m_i} \frac{a_\alpha}{a_e} V_{\alpha}$, we find the final form of the equations, which describes the dynamical (temporal) part of our problem

$$\frac{d^2 V_{\alpha}}{d t^2} + \eta V_{\alpha} + p \left( V_{\alpha} + \frac{m_\alpha}{m_i} V_{\alpha} \right) \left( \frac{e_{\alpha_0}}{e_{\alpha_0}} \right) = 0 \quad (3)$$

Where, $e_\alpha = n_{0b} / n_0$ and $\eta = \gamma k^2 V_{\alpha_0}^2$.

Here we shall confine our analysis by two important cases:

1) HF stabilization of two-stream (Buneman) instability and
2) the influence of the HF electric field on the dispersion characteristics of unstable surface waves excited in a warm plasma waveguides by an electron beam [7].

III. SUPPRESSION OF BUNEMAN INSTABILITY BY MEANS OF HF FIELD (CYLINDRICAL GEOMETRY).

Let us suppose that the electron component moves relative to the ion component with velocity

$$\vec{u}_0 = \vec{u}_e - \vec{u}_i \quad (e_b = n_{0b} / n_0 = 0 - \text{there is no externally injected beam}.)$$

For characteristic frequency of HF field much greater than the natural (eigen) frequency of excited surface waves ($\omega_0 >> \omega_{sw} \approx \omega_p$), we may employ method of averaging.

Using Jacobi-Anger formula:

$$e^{i \omega_0 t} = \sum_{m=-\infty}^{\infty} J_m (a) e^{i \omega_0 t},$$

Where $J_m (a)$ - the Bessel functions, for values
<v_{a_i}>=\omega_{a_i} \int_{0}^{2\pi/m_{a_i}} v_{a_i}(t) dt
from equation (1) we have
\frac{\partial^2 v_{a_i}}{\partial t^2} + \eta_i v_{a_i} + p_i^2 (v_{a_i}) + e^{i(k_u r)} J_0(a) <v_{a_i}> = 0 \quad (4)
\frac{\partial^2 v_{\phi}}{\partial t^2} + \frac{m_e}{m_i} p_i^2 (v_{\phi}) + e^{i(k_u t)} J_0(a) <v_{\phi}> = 0
The system of equations (4) coincides (within the redefinition of \omega_p^2 \rightarrow p_i^2, \omega_p^2 \rightarrow (m_e/m_i) p_i^2, V_u = 0 in cold plasma) with the system describing HF suppression of Buneman instability in a uniform (or nonuniform) unbounded (or bounded) plasma [21] or [22].

A. Solution of "temporal" (time-dependent) equations
Following the procedure, developed in [9], from equations (4) we derive the dispersion equation
\begin{equation}
1 = \omega_p^2 \left( \frac{\omega - k_u \lambda^2}{\omega - k_u \omega_p^2} \right) + \frac{\eta_i}{\omega - k_u \omega_p^2} \left( 1 - \omega_p^2 \right) J_0^2(a) / \omega^2 \left( \frac{\omega - k_u \omega_p^2}{\omega - k_u \lambda^2} \right) \end{equation}
In the case when \bar{E}_u = 0 and V_u = 0, then equation (5) agrees with the dispersion equation, which describes the unstable oscillations that are excited in uniform unbounded (bounded) plasma by a low density electron beam [21,22], we shall analyze equation (5) in two cases:

A.1. "Nonresonant" instability (\omega << |k u_0|).
We have from equation (5)
\begin{equation}
\omega^2 = \frac{\omega_p^2}{} \left( 1 - \frac{1}{k u_0} \left( \frac{\eta_i + \omega_p^2}{k u_0} - \omega_p^2 \right) \right) \end{equation}
Equation (6) shows that \omega^2 are negative, i.e., small perturbations grow exponentially in time if the following condition is satisfied
\omega^2 > \frac{1}{(k u_0)^2} \left( 1 - \frac{1}{k u_0} \left( \frac{\eta_i + \omega_p^2}{k u_0} - \omega_p^2 \right) \right) > p_i^2 (1 - J_0^2(a)) \end{equation}
The roots of equation (6) under the above condition are complex and one of them corresponds to instability with the growth rate
\begin{equation}
\gamma_b = \gamma_b \left[ 1 - \frac{1}{(k u_0)^2} \left( \frac{\eta_i + \omega_p^2}{k u_0} - \omega_p^2 \right) \right] \frac{p_i^2 (1 - J_0^2(a))}{} \end{equation}
where \gamma_b is the growth rate at \bar{E}_u = 0 (a = 0):
\begin{equation}
\gamma_b = \omega_p^2 \left[ \frac{1}{k u_0} \left( \frac{\eta_i + \omega_p^2}{k u_0} - \omega_p^2 \right) \right]^{1/2} \end{equation}
At \bar{V}_u = 0 the result agrees with the result for cold plasma [1,2].

A.2. "Resonant" instability (\omega u_0 \approx p).
The growth rates at \bar{E}_u = 0 reached its maximum value under the condition \omega u_0 \approx p (see [3]). Accordingly, the dispersion equation (5) will have the form
\begin{equation}
\omega^2 \left( \frac{\omega - k_u \lambda^2}{\omega - k_u \omega_p^2} \right) - \omega_p^2 \left( \frac{\eta_i - \omega_p^2}{k u_0} \right) = 0 \end{equation}
The frequency and the growth rate of unstable surface waves in the resonant case are determined by the following expressions:
\begin{equation}
(\text{Re}\omega)^{HF}_{Rw} = - \frac{1}{4} \left[ \frac{\eta_i}{\omega - k_u \omega_p^2} - \frac{p}{m_i} \right] \end{equation}
\begin{equation}
(\text{Im}\omega)^{HF}_{Rw} = \sqrt{\frac{3}{2}} p \left[ \frac{\eta_i}{\omega - k_u \omega_p^2} - \frac{p}{2m_i} \right]^{1/3} \end{equation}
where p is determined by the equation of the space part of the problem.

A comparison of the expression (11) for \gamma'_{Rw} with the growth rate \gamma'_{Rw} (a = 0) and \eta = 0 shows that application of the HF electric field reduces the growth rate by a factor \omega_{J_0}^{2/3} in cold plasma.

The results obtained are in full agreement with the conclusion that an external HF field may have a stabilizing effect on the Buneman instability in uniform (or nonuniform) plasma [20] or [21]. The warmness plasma reduced the growth rate.

We conclude that the growth rate of a Buneman instability decreased more in a relativistic warm plasma than in nonrelativistic warm (or cold) plasma which have been considered by Demchenko et al. [2,20,21].

B. Solution of the space (spatial) part of the problem
The main feature of the expression (9), (10) and (11) consists in an existence of a separatrix constant p, which enables us to consider the plasma boundaries.

To find an explicit expression for the constant p it is necessary to solve the following differential equation (for details see [8]):
\begin{equation}
\frac{1}{r} \frac{d}{dr} \left[ r \varepsilon(p,r) \frac{d}{dr} \Phi_p(r) \right] - \left( \frac{m_i}{r^2} + k^2 \right) \varepsilon(p,r) \Phi_p(r) = 0 \end{equation}
where \varepsilon(p,r) = 1 - \omega_p^2(r)/p^2. If the radial profile of
the plasma density and boundary conditions are specified, the solution of equation (12) gives us desired value of the separation constant $p$. The feature of the equation (12) is that neither the amplitude of HF electric field nor electron beam parameters enters into it. Therefore equation (12) coincides with equation describing propagation of natural (free of external influence) electrostatic surface waves in a nonuniform plasma cylinder (see e.g., [20] and ref. therein). Supposing that plasma density is uniform and the interface between plasma and vacuum is sharp.

In such case the relations determine the solution of equation (12)

$$\Phi_2 (r < R) = C_1 I_m (kR), \Phi_2 (r > R) = C_2 K_m (kR), (13)$$

where $I_m (kR)$ and $K_m (kR)$ are the modified Bessel functions and $C_{1,2}$ are the constants. Using the continuity condition of $\Phi_2$ and $\varepsilon d \Phi_2 / d r$ at the boundary, the following equation is found (see e.g. [20] and refs. therein)

$$\varepsilon_0 (p) + \eta_m (kR) = 0, (14)$$

where the stroke means differentiation with respect to the argument. Equation (14) gives us the relation between the separation constant $p$, electron plasma frequency $\omega_p$ and the axial wave number $k$:

$$p = \omega_p (kR)^{1/2} (K_m (kR) I'_m (kR))^{1/2}. (15)$$

In the limiting case of small radius of plasma waveguide $(kR << 1)$, from (16) we find

$$p_{m=0} = \omega_p (kR)[\frac{1}{2} \ln \frac{kR}{2}]^{1/2}, \quad p_{m=0} \approx \omega_p \frac{kR}{\sqrt{2}}. (16)$$

For $kR >> m$ ("thick" waveguide) always $\eta_m (kR) \approx 1$ and we have $p = \omega_p / \sqrt{2}$.

C. Suppression of Buneman instability (plane geometry).

The main feature of the expressions (9), (10), and (11) consists of an existence of a separation constant $p$, which enables us to taken into account the plasma boundaries.

To find an explicit expression for the constant $p$ it is necessary to solve the following differential equation (for detail see [8]):

$$\frac{d}{d x} \left( \varepsilon (p, x) \frac{d \Phi_2}{d x} \right) - k^2 \varepsilon (p, x) \Phi_2 = 0, (17)$$

where $\varepsilon (p, x) = 1 - \omega_p^2 (x) / p^2$. The solution of equation (17) in the different regions has the form

$$\Phi_2 (x) = c_1 e^{kx} + c_2 e^{-kx} \quad (-l/2 < x < l/2) (18)$$

where $c_j (j = 1, ..., 4)$ are the integration constants.

Using the continuity condition of $\Phi_2$ and $\varepsilon d \Phi_2 / d r$ at the points $x = \pm l/2$, we get

$$\eta_m (x) = \frac{2 \varepsilon_0 (p)}{\varepsilon_0 (p) + 1}. (19)$$

Equation (20) can be satisfied only for $\varepsilon_0 (p) < 0$ ($\omega_{pe} > p$). In the case of a "thick" plasma layer ($kl >> 1$), as would be expected, equation (20) coincides with the dispersion equation $\varepsilon_0 (p) = -1$ for surface oscillations of a semi - bounded plasma $(p = \omega_p / \sqrt{2})$. For a "thin" plasma layer ($kl << 1$), equation (20) is solvable only if $\varepsilon_0 > 1$. Then it takes the form $(kl) \varepsilon_0 (p) = -2$, which gives the relation between constant $p$, electron plasma frequency $\omega_p$ and the wave number $k$:

$$p = \omega_p (kl)^{1/2}. (21)$$

The dispersion equation describing the electron-ion (Buneman) instability in a plane warm plasma waveguide coincides with equation (5), where the separation constant $p$ is now determined by equation (20).

This enables us to conclude that independent of the geometry of the problem the conclusion on the HF stabilization of Buneman instability in a warm plasma waveguide remains valid.

This equation is the same equation in the previous work [8,19,20] i.e., the warm plasma waveguide has no effect on the space part of the problem.

IV. THE INFLUENCE OF HF ELECTRIC FIELD ON THE INSTABILITY OF A LOW - DENSITY ELECTRON BEAM PASSING THROUGH WARM PLASMA WAVEGUIDE

Let us now assume that an electron beam of low density ($\varepsilon_b = (n_{0i} / n_0) \ll 1$) is passing through quasineutral warm plasma with the velocity $\tilde{u}_{b0}$. We shall also suppose that both plasma components are at rest ($\tilde{u}_{e0} = \tilde{u}_{b0} = 0$). According to equations (3), plasma oscillations are then described by the dispersion equation

$$(\omega^2 - \omega_{1f}^2) [(\omega - k \tilde{u}_{e0})^2 (\omega^2 - \omega_{1f}^2 - \eta) - \varepsilon_y (\omega^2 - \eta)] = 0, (22)$$

where

$$\omega_{1f}^2 (p) = \frac{m_e}{m_i} p^2 (1 - J_0^2 (a)), \quad \omega_{1f}^2 (p) = p^2 (1 + \frac{m_i}{m_e} J_0^2 (a)) (23)$$

In the case of vanishing electric field amplitude ($\tilde{E}_0 = 0$),
then equation (22) agrees with the dispersion equation which describes the unstable oscillations that excited in a uniform unbounded (bounded) plasma by a low - density electron beam [2,3]. Same as in [2,3] we shall analyze equation (22) in two cases:

\[ \omega = k u_b \pm \sqrt{\varepsilon_0 p (k u_b (k u_b - \eta))^3} / [(k u_b)^2 - \omega_{HF}^2 (p - \eta)]^{1/2}. \]  
(24)

Under the conditions
\[ p^2 (1 + m_e J_0^2 (a)) > k u_b (k u_b - \eta) > 0, \]  
(25)
the roots of equation (24) are complex and one of them corresponds to instability with the growth rate
\[ \gamma_{NR} = \sqrt{\varepsilon_0 p (k u_b (k u_b - \eta))^3} / [(\omega_{HF} (p - k u_b (k u_b - \eta)))]^{1/2}. \]  
(26)

At \( V_{th} = 0 \) the result agrees with the result for cold plasma [2].

**B. Resonant case (k \( u_b \approx \omega_{HF} \))**

The frequency of unstable oscillations can be represented in the form \( \omega = \omega_{HF} (p) + \Delta \omega \), where
\[ \varepsilon_{1/3} p (1 + m_e J_0^2 (a))^{1/6} \]
\[ \Re \Delta \omega = - \frac{2^{4/3} [1 + \frac{2 \eta}{m_i} (p^2 (1 + m_e J_0^2 (a)))^{1/2}]^{1/3}}{2^{4/3} [1 + \frac{2 \eta}{m_i} (p^2 (1 + m_e J_0^2 (a)))^{1/2}]^{1/3}}, \]  
(27)
and
\[ \gamma_r = \Im \Delta \omega = \varepsilon_{1/3} p (1 + m_e J_0^2 (a))^{1/6} \]
\[ \frac{2^{4/3} [1 + \frac{2 \eta}{m_i} (p^2 (1 + m_e J_0^2 (a)))^{1/2}]^{1/3}}{2^{4/3} [1 + \frac{2 \eta}{m_i} (p^2 (1 + m_e J_0^2 (a)))^{1/2}]^{1/3}}. \]  
(28)

where \( p \) is determined by the equation of the space part of the problem.

It follows from expressions (24) - (28) that the HF electric field has no essential influence on the dispersion characteristics of unstable surface waves excited in a plasma waveguide by a low - density electron beam. The region of instability only slightly narrows and the growth rate decreases by a small parameter.

The results obtained are in a full agreement with the conclusion that an external HF field may have a stabilizing effect on the electron beam-plasma interaction in uniform (or nonuniform) plasma [2,3]. The warm plasma reduced the growth rate.

We conclude that the growth rate of the electron beam-plasma interaction decreases more in warm plasma than in cold plasma [2].

**V. CONCLUSIONS**

The paper deals with parametric excitation of the potential surface waves in bounded nonuniform warm plasma by a monochromatic HF electrical field. It is shown that the problem can be reduced to the solution of the “temporal” (parametric) and “stationary” (spatial) parts. The “temporal” part that determines the frequencies and growth rates of unstable oscillations coincides with the accuracy to redefine the natural frequencies with the equations that describes the parametric resonance in homogeneous plasma. The natural frequencies of oscillations and the spatial distribution of the amplitude of the self-consistent electric field are determined from the solution of a boundary-value problem ("space" part) taking into account a specific spatial distribution of the plasma density.

The application of the HF electric field reduces the growth rate by a factor \( J_0^{1/3} \) in cold plasma. Regardless the geometry of the problem the application of the HF filed may stabilize the Buneman instability in a warm plasma waveguide. The HF electric field has no essential influence on the dispersion characteristics of unstable surface waves excited in a plasma waveguide by a low-density electron beam. The region of instability only slightly narrows and the growth rate decreases by a small parameter. We show also that the growth rate of the Buneman instability and the electron beam-plasma interaction decrease in warm plasma more than in cold plasma. The plasma electrons have no influence on the solution of the space part of the problem.

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