Estimation of Probable Maximum Precipitation Using Statistical Methods

N. Vivekanandan

Abstract **- Probable Maximum Precipitation (PMP) is generally used to arrive at estimates for Probable Maximum Flood (PMF) for planning, design and risk assessment of high hazard hydrological structures. PMP represents the greatest depth of precipitation for a given duration which is physically possible over a given size storm area at a particular geographical location and at a particular time of year. Number of methods such as statistical, empirical and dynamic is generally used for estimation of PMP for non-orographic or plain area regions. In this paper, statistical method is used for estimation of PMP. Extreme Value Type-1 distribution (EV1) is adopted to estimate the extreme rainfall and the results are compared with the 1-day PMP obtained from Hershfield method. Anderson-Darling test is applied for checking the adequacy of fitting of EV1 distribution to the series of annual 1-day maximum rainfall data. Wald-Wolfowitz run test and Mann-Whitney Wilcoxon U-test is performed on the rainfall data to check the randomness and homogeneity of the data series. Grubbs test is applied to identify the outliers in the data series. The study suggests the estimated 1-day PMP viz., 25.7 cm for Devarapalle and 46.3 cm for Visakhapatnam could be used to arrive at PMF estimates for design purposes.**

Index Terms: **Anderson-Darling, Extreme Value Type-1, Hershfield, Probable Maximum Precipitation, Rainfall**

I. INTRODUCTION

Probable Maximum Precipitation (PMP) is theoretically defined as the greatest depth of precipitation for a given duration which is physically possible over a given size storm area at a particular geographical location and at a particular time of year [1]. Hydrologists use a PMP magnitude together with its spatial and temporal distributions for the catchments of a dam to calculate the Probable Maximum Flood (PMF). In the case where the risk of a dam overtopping is deemed unacceptable, an estimate of the PMP depth is used to determine the PMF for that location. Moreover, generation of the PMF using PMP has become as a standard method for dam design in many parts of the world. Since 1950, a number of methods have been developed for estimating PMP including the United States, China, India, and Australia [2]. The National Weather Service (NWS) has published and updated numerous hydrometeorological reports for estimating the PMP in different regions of the United States.

Methods available for estimation of PMP non-orographic or plain area regions are classified into three categories, viz., statistical method, empirical method and dynamic method. Statistical method utilizing data of long period for rain gauge stations. This method is particularly useful for making quick estimates for basins having size less than 1000 km^2 [3]. A major shortcoming in this method is that it yields only point values of PMP and thus requires area reduction curves for adjusting the point values to various size areas. Statistical method involves statistical analysis of station observations on extreme rainfall, which can be employed wherever sufficient precipitation data are available, and are particularly useful where other such meteorological data as dew point and wind records are not available. Empirical method is based on time series analysis using rainfall data whereas dynamic method is based on rain storm analysis and their transposition over the basin coupled with moisture maximization. In addition to above, Hershfield method based on statistical concepts is also generally used to determine the PMP value.

During the past, a number of studies on PMP estimation have been carried out by different researchers Ghahraman and Sepaskhah [4] made some modifications in the suggested model developed by Bethlahmy [5] and offered a new method for estimating extreme rainfall values for the southern parts of Iran. They showed that values calculated on the basis of Bethlahmy [5] and Hershfield [6] have significant difference from values by synoptical estimated method. Koutsoyiannis [7] developed a rather straightforward method for assigning a return period to PMP values estimated using the frequency factor method.

Foufoula-Georgiou [8] investigated a storm transposition approach for assessing the frequency of extreme precipitation depths, but stressed the need for further research before applying the method to the PMP and PMF. Paimozd [9] applied synoptical and statistical methods for estimation of PMP at eastern basins of Hormozgan province. He found that the PMP estimated from Hershfield method resulted in estimating larger values in comparison to the synoptical method but the values calculated from Hershfield method were closer to synoptical method. This paper presents a study on estimation of PMP adopting Extreme Value Type-1 (EV1) distribution and Hershfield method with illustrative example.

II.METHODOLOGY

The study is to estimate PMP for Devarapalle and Visakhapatnam adopting statistical method. Thus, it is required to process and validate the data series such as (i) Check the randomness, homogeneity and outliers in the data series using statistical tests; (ii) compute the extreme rainfall by EV1 distribution; (iii) assess the adequacy of fitting of EV1 distribution to the data series using Anderson-Darling (A^2) test; (iv) estimate the PMP by Hershfield method; and (v) analyse the results obtained thereof.

N. Vivekanandan, Central Water and Power Research Station, Pune 411024, Maharashtra, E-mail: anandaan@rediffmail.com

A) Extreme Value Type-1 Distribution

The Probability Density Function (PDF) and Cumulative Distribution Function (CDF) of EV1 distribution is given by:

$$\left.\begin{aligned} \text{PDF}: f(R;\alpha,\beta) &= \frac{e^{-(R-\alpha)/\beta} e^{-e^{-(R-\alpha)/\beta}}}{\beta}, R_i, \beta > 0 \\ \text{CDF}: F(R;\alpha,\beta) &= e^{-e^{-(R-\alpha)/\beta}} \text{ with } R = (R_1, R_2, \ldots, R_N) \end{aligned}\right\} \quad \ldots (1)$$

where α and β are location and scale parameters of the distribution [10]. The extreme rainfall (R_T) for a given return period (T) is computed using $R_T = \alpha + Y_T\beta$ where $Y_T = -\ln(-\ln(1-(1/T)))$.

B) Goodness-of-Fit (GoF) test

Generally, A^2 test [11] is applied for checking the adequacy of fitting of EV1 distribution to the series of rainfall data. The A^2 statistic is defined by:

$$A^2 = (-N) - (1/N)\sum_{i=1}^{N}\{(2i-1)\ln(Z_i) + (2N+1-2i)\ln(1-Z_i)\} \quad \ldots (2)$$

Here, $Z_i = F(R_i)$ for i=1,2,3,…,N with $R_1 < R_2 < \ldots R_N$, $F(R_i)$ is the CDF of i^{th} sample (R_i) and N is the sample size.

C) Hershfield Method

Hershfield [12] proposed a method using the parameters viz., mean, standard deviation and the frequency factor for the series of annual maximum rainfall. Thus, the mathematical representation of Hershfield method for the PMP of any duration (i.e., 1-day, 2-day and 3-day) can be estimated from:

$$PMP = \overline{R_N} + K_M S_N \quad \ldots (3)$$

where, PMP is the estimated 1-day PMP from Eq. (3), $\overline{R_N}$ and S_N is the mean and standard deviation of Annual 1-Day Maximum Rainfall (ADMR) series has N observations and K_M is the frequency factor which depends upon the number of observations. The value of K_M [13] can be obtained from:

$$K_M = \frac{R_{MAX} - \overline{R_{N-1}}}{S_{N-1}} \quad \ldots (4)$$

where R_{MAX} is the highest value in the series of ADMR, $\overline{R_{N-1}}$ and S_{N-1} is the mean and standard deviation of ADMR series has $N-1$ observations excluding the highest value of R_{MAX}.

III. APPLICATION

In this paper, a study on estimation of PMP was carried out for Devarapalle and Visakhapatnam. The series of ADMR was derived from the daily rainfall data and used for estimation of 1-day maximum rainfall (also referred as extreme rainfall) adopting EV1 distribution and 1-day PMP using Hershfield method. The rainfall data recorded at Devarapalle for the period 1999 to 2012 and Vishakhapatnam for the period 1973 to 2012 was used. Table 1 gives the descriptive statistics of the series of ADMR recorded at these rain gauge stations.

Table 1: Descriptive statistics of ADMR

Descriptive statistics	Devarapalle	Visakhapatnam
Average (mm)	97.6	128.3
Standard deviation (mm)	39.5	58.0
Skewness	1.427	2.072
Kurtosis	2.530	6.869

IV. RESULTS AND DISCUSSIONS

By applying the procedures, as described above, a computer code was developed in FORTRAN language and adopted to estimate the extreme rainfall adopting EV1 distribution and compute PMP using Hershfield method. The program also computes the statistical tests results for both Devarapalle and Visakhapatnam.

A) Data Validation

The hydrologic data used for frequency analysis should be independent and identically distributed with the hydrologic system producing the phenomenon considered say, rainfall (or) stream flow is to be random in nature, as also independent in space and time. Similarly, homogeneity of the sample elements in the data series has to be checked to identify whether the data originates from a single population or not. The presence of outliers in a data sample has undesirable impacts on frequency analysis and thus, the sample also needs to be checked for outliers if any.

For the present study, Wald-Wolfowitz run test and Mann-Whitney Wilcoxon U-test [14] was performed on the rainfall data to check the randomness and homogeneity of the data series of ADMR. In addition to above, the Grubbs test was applied to identify the outliers in the data series. The statistical test results were presented in Table 2.

Table 2: Test results of randomness and homogeneity

Data series	Wald-Wolfowitz (Test for randomness)		Mann-Whitney-Wilcoxon (Test for homogeneity)	
	Computed value	Critical value	Computed value	Critical value
Devarapalle	0.943	1.96	0.037	1.96
Visakhapatnam	0.161	1.96	0.326	1.96

From Table 2, it may be noticed that the computed values of Wald-Wolfowitz and Mann-Whitney-Wilcoxon test statistics for series of ADMR pertaining to Devarapalle and Visakhapatnam rain gauge stations are not greater than the critical value at 5% level; and at this level, the series of ADMR was found to be random and also homogeneous. Based on Grubb's outlier test results, ADMR in respect the observed maximum rainfall of 200 mm (2012) at Devarapalle and 371.2 mm (1982) at Visakhapatnam were noted to be outliers. However, based on the hydrologic judgment of the study region, the entire data set relating to the series of ADMR was used for estimation of PMP.

B) Estimation of Extreme Rainfall using EV1 distribution

Extreme value analysis of ADMR obtained from Devarapalle and Visakhapatnam rain gauge stations were carried out with EV1 distribution. The parameters were determined by maximum likelihood method and used for estimation of extreme rainfall. Table 3 gives the extreme rainfall estimates with standard error for different return periods for both Devarapalle and Visakhapatnam. In this context, AERB [15] guidelines stated that the Mean+SE (where Mean denotes the estimated extreme rainfall and SE the Standard Error) value is generally used for arriving at a design value.

Table 3: Extreme rainfall estimates with standard error using EV1

Return period (year)	Devarapalle		Visakhapatnam	
	ER (mm)	SE (mm)	ER (mm)	SE (mm)
2	91.4	9.3	118.8	8.3
5	125.0	15.7	169.4	14.0
10	147.2	21.2	202.9	18.9
20	168.6	26.8	235.1	23.9
25	175.3	28.6	245.3	25.5
50	196.2	34.2	276.7	30.5
100	216.9	39.9	307.8	35.5
200	237.5	45.5	338.9	40.6
250	244.2	47.4	348.9	42.2
500	264.7	53.1	379.9	47.3
1000	285.3	58.8	410.9	52.5
ER: Estimated Rainfall; SE: Standard Error				

The estimated extreme rainfall obtained from EV1 distribution was used to develop the rainfall frequency curves and presented in Figures 1 and 2.

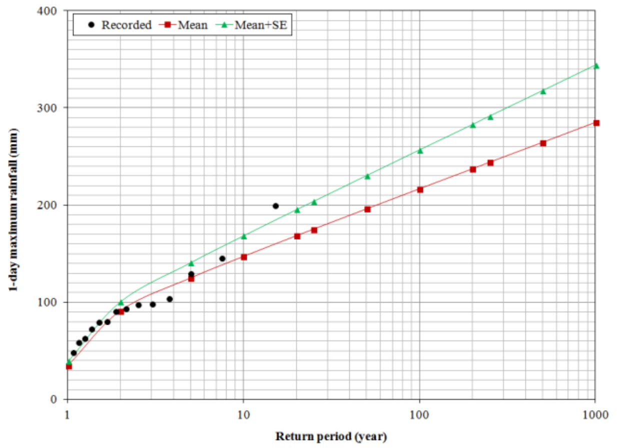

Figure 1: Plots of recorded and estimated 1-day maximum rainfall using EV1 distribution for Devarapalle

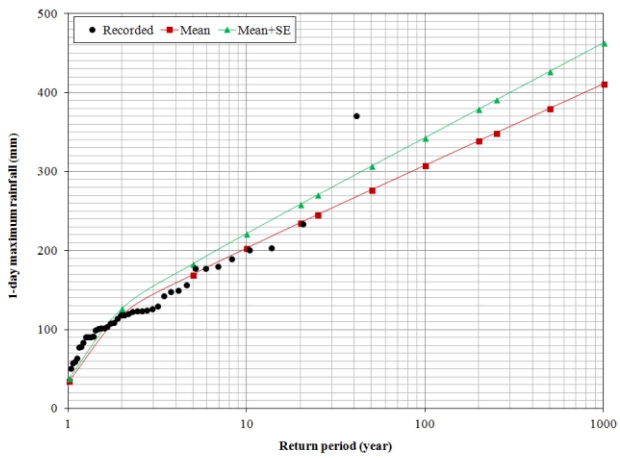

Figure 2: Plots of recorded and estimated 1-day maximum rainfall using EV1 distribution for Visakhapatnam

C)Analysis based on GoF test

A^2 test statistic values adopting EV1 distribution for Devarapalle and Visakhapatnam stations were computed as 0.202 and 0.501 respectively. The test results showed that the computed values are not greater than the theoretical value of 0.757 at 5% level, and at this level, EV1 distribution is found to be acceptable for modelling the ADMR.

D)Estimation of PMP using Hershfield Method

For Hershfield method, the parameters such as R_{MAX}, $\overline{R}_N$, S_N, $\overline{R}_{N-1}$, S_{N-1} and K_M were computed from the series of ADMR and used for estimation of PMP. Table 4 gives the computed values of the parameters used for estimation of PMP for the stations under study.

Table 4: Parameters used in estimation of PMP

Parameters	Devarapalle	Vishakhapatnam
R_{MAX} (mm)	200.0	371.2
$\overline{R}_N$ (mm)	97.6	128.3
S_N (mm)	39.5	58.0
$\overline{R}_{N-1}$ (mm)	89.7	122.0
S_{N-1} (mm)	27.3	43.1
K_M	4.032	5.778
1-day PMP (mm)	256.8	463.4

V. CONCLUSIONS

The paper described the procedures adopted in estimation of extreme rainfall adopting EV1 distribution and PMP using Hershfield method. From the results of data analysis, the following conclusions were drawn from the study:

i) Wald-Wolfowitz and Mann-Whitney-Wilcoxon test results indicated that the series of ADMR in respect of Devarapalle and Visakhapatnam is found to be random and also homogeneous.

ii) Grubb's outlier test results showed that the ADMR in respect the observed maximum rainfall of 200 mm at Devarapalle and 371.2 mm at Visakhapatnam are noted to be outliers. However, based on the hydrologic judgment of the study region, the entire data set relating to the series of ADMR was used for rainfall frequency analysis.

iii) The A^2 test results supported the use of EV1 distribution for modelling the ADMR.

iv) For Devarapalle, the 100-year and 1000-year return period Mean+SE values of extreme rainfall adopting EV1 distribution was computed as 256.8 mm and 344.1 mm respectively. For Vishakhapatnam, Mean+SE values were computed as 343.3 mm for 100-year whereas 463.4 mm for 1000-year.

v) By using Hershfield method, the 1-day PMP for Devarapalle and Visakhapatnam regions were computed as 256.8 mm and 463.4 mm.

vi) For Devarapalle, it was found that the 100-year return period extreme rainfall of 256.8 mm obtained from EV1 distribution is equal to the estimated 1-day PMP using Hershfield method

vii) For Visakhapatnam, it was observed that the estimated 1-day PMP of 463.4 mm obtained from Hershfield method is equal to 1000-year return period extreme rainfall given by EV1 distribution.

viii) The 1-day rainfall depth for Devarapalle and Visakhapatnam regions were found to be 25.7 cm and 46.3 cm respectively.

ix) The results presented in the paper would be helpful to the stakeholders to arrive at PMF for planning, design and risk assessment of high hazard hydrological structures in the regions.

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N. Vivekanandan post graduated in mathematics from Madurai Kamaraj University in 1991. He obtained Master of Engineering in hydrology from University of Roorkee in 2000. He obtained Master of Philosophy in mathematics from Bharathiar University in 2006 and MBA (Human Resources) from Manonmaniam Sundaranar University in 2013. From 1993 (May) to 2006 (March), he worked as Research Assistant in Central Water and Power Research Station (CWPRS), Pune. From 2006 (April) to till date, he is working as Assistant Research Officer in CWPRS wherein conducting hydrological and hydrometeorological studies using probabilistic and stochastic models for various water resources projects.